Wentao Lu

1. a) $A = \begin{pmatrix} 1&2&3&4 \\ 1&2&3&4 \end{pmatrix} \begin{pmatrix} 1&2&3&4 \\ 1&3&4&2 \end{pmatrix} \begin{pmatrix} 1&2&3&4 \\ 2&1&4&3 \end{pmatrix} \begin{pmatrix} 1&2&3&4 \\ 3&4&1&2 \end{pmatrix}$

$\begin{pmatrix} 1&2&3&4 \\ 4&3&2&1 \end{pmatrix} \begin{pmatrix} 1&2&3&4 \\ 2&3&1&4 \end{pmatrix} \begin{pmatrix} 1&2&3&4 \\ 3&2&1&4 \end{pmatrix} \begin{pmatrix} 1&2&3&4 \\ 1&3&4&2 \end{pmatrix}$

$\begin{pmatrix} 1&2&3&4 \\ 3&2&4&1 \end{pmatrix} \begin{pmatrix} 1&2&3&4 \\ 2&4&3&1 \end{pmatrix} \begin{pmatrix} 1&2&4&3 \\ 1&4&3&2 \end{pmatrix} \begin{pmatrix} 1&2&4&3 \\ 4&1&2&3 \end{pmatrix}$

10 elements

Let $G = S_4$ $H = A_4$

since A_4 is set of all even permutation of S_4

Assume $P_1 \cdot P_2 \in A_4$ is two permutation of A_4

$P_1 \cdot P_2^{-1} \in A_4$

so A_4 is subgroup of S_4

b) $H = \left\{ \begin{pmatrix} 1&2&3&4 \\ 1&2&3&4 \end{pmatrix} \begin{pmatrix} 1&2&3&4 \\ 2&3&1&4 \end{pmatrix} \begin{pmatrix} 1&2&3&4 \\ 3&1&2&4 \end{pmatrix} \right\}$

$= \{ I, (123), (132) \}$ in cycle form

H is non empty and $I \cdot (123) = (123) \cdot I \in H$

so H is subgroup of G

c) index of H in A_4 is 4

$I(H) = H$

$(234)H = \{(234), (142), (13)(24)\}$

$(13)(24)H = \{(13)(24), (234), (142)\}$

$(14)(23)H = \{(14)(23), (24), (134)\}$

d) $|H| = 3$ A_4 is 12

so it always be 3 divides 12

2. if H is a subgroup of a group G then $[G] = [G:H] \cdot |H|$