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Homework 16

(1) Consider the alternating group called A_4 on $\{1, 2, 3, 4\}$ i.e. the group of all permutations with an even number of inversions

(a) List them all (in lexigraphical order of bottom line)

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$$

A_4 has 4 members in the subgroup.

(b) Consider the following set H.

$$H = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix} \right\}$$

Show H is a subgroup of A_4

$H \subseteq A$ and has the same properties as A_4.

(c) Find all the left cosets of H in A_4. How many are there?

two cosets. There would be 3 cosets.

(d) Explain why this example illustrates the famous Lagrange theorem that for any finite group G and any of its subgroup H, $|G|/|H$ is always an integer.

Since $H \subseteq G$, $|G|/|H|$ would always be an integer.

(2) If H is a subgroup of G, then $G = n|H|$ for $\exists n \in \mathbb{N}$.

Proof. Take any $r_1 \in G$. Note $|Hr_1| = |H|$. If $Hr_1 \neq G$, then take any $r_2 \in G \setminus Hr_1$. H_1 and H_2 are disjoint so $|Hr_1 \cup Hr_2| = 2|H|$.
By continuing this, we have some $n \in \mathbb{N}$ s.t. $|G| = n|H|$ and $G = Hr_1 \cup \cdots \cup Hr_n$.