Sarah Calovic Homework 16 11/14/2021

1. a) $\begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix}$ $\begin{pmatrix}1&2&3&4\\1&3&4&2\end{pmatrix}$ $\begin{pmatrix}1&2&3&4\\1&4&2&3\end{pmatrix}$ $\begin{pmatrix}1&2&3&4\\2&1&4&3\end{pmatrix}$ $\begin{pmatrix}1&2&3&4\\2&3&1&4\end{pmatrix}$

$\begin{pmatrix}1&2&3&4\\2&4&3&1\end{pmatrix}$ $\begin{pmatrix}1&2&3&4\\3&1&2&4\end{pmatrix}$ $\begin{pmatrix}1&2&3&4\\3&2&4&1\end{pmatrix}$ $\begin{pmatrix}1&2&3&4\\3&4&1&2\end{pmatrix}$ $\begin{pmatrix}1&2&3&4\\4&1&3&2\end{pmatrix}$ $\begin{pmatrix}1&2&3&4\\4&2&1&3\end{pmatrix}$

$\begin{pmatrix}1&2&3&4\\4&3&2&1\end{pmatrix}$

A_4 has 12 elements

A_4 has the identity element $\begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix}$. The product of any two permutations is in A_4, since both have an even number of inversions, so their product would have an even x even = even number of inversions.

b) $\begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix}\begin{pmatrix}1&2&3&4\\2&3&1&4\end{pmatrix}=\begin{pmatrix}1&2&3&4\\2&3&1&4\end{pmatrix}$ $\begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix}\begin{pmatrix}1&2&3&4\\3&1&2&4\end{pmatrix}=\begin{pmatrix}1&2&3&4\\3&1&2&4\end{pmatrix}$

$\begin{pmatrix}1&2&3&4\\2&3&1&4\end{pmatrix}\begin{pmatrix}1&2&3&4\\3&1&2&4\end{pmatrix}=\begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix}$ so the products of each element with each other is in the subgroup H. H contains the identity element $\begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix}$, so it is a subgroup.

c. $\begin{pmatrix}1&2&3&4\\1&3&4&2\end{pmatrix} * H = \left\{ \begin{pmatrix}1&2&3&4\\1&3&4&2\end{pmatrix}, \begin{pmatrix}1&2&3&4\\2&1&4&3\end{pmatrix}, \begin{pmatrix}1&2&3&4\\3&2&4&1\end{pmatrix} \right\}$

$\begin{pmatrix}1&2&3&4\\1&4&2&3\end{pmatrix} * H = \left\{ \begin{pmatrix}1&2&3&4\\1&4&2&3\end{pmatrix}, \begin{pmatrix}1&2&3&4\\2&4&3&1\end{pmatrix}, \begin{pmatrix}1&2&3&4\\3&4&1&2\end{pmatrix} \right\}$

$\begin{pmatrix}1&2&3&4\\4&1&3&2\end{pmatrix} * H = \left\{ \begin{pmatrix}1&2&3&4\\4&1&3&2\end{pmatrix}, \begin{pmatrix}1&2&3&4\\4&2&1&3\end{pmatrix}, \begin{pmatrix}1&2&3&4\\4&3&2&1\end{pmatrix} \right\}$

And H itself is its own coset. They have the same number of elements in each because to get the left cosets multiply the element of A_4 by each element in H, so they would all have the same number. They cannot have any common elements because if they did, they would be the same set.

d. $\frac{|G|}{|H|}$ $\frac{12}{3} = 4$

2. For any group G and subgroup H, $\frac{|G|}{|H|}$ is an integer.