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Okay

H.W 6

1a. $\begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\1&3&4&2\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\1&4&2&3\end{pmatrix}$,
$\begin{pmatrix}1&2&3&4\\2&1&4&3\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\2&3&1&4\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\2&4&3&1\end{pmatrix}$,
$\begin{pmatrix}1&2&3&4\\3&1&2&4\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\3&2&4&1\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\3&4&1&2\end{pmatrix}$,
$\begin{pmatrix}1&2&3&4\\4&1&3&2\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\4&2&1&3\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\4&3&2&1\end{pmatrix}$

12 members in A_4.
$|a*H| = |b*H|$ so number of even and odd inversion is the same that add up to 24.

1b. $H = \left\{ \begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix}, \begin{pmatrix}1&2&3&4\\2&3&1&4\end{pmatrix}, \begin{pmatrix}1&2&3&4\\3&1&2&4\end{pmatrix} \right\}$

Notice that $\begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix}$ "multiplied"

with any other member in H is going to give back the member thus we have an identity element. Also if you "multiply" the last two members you get the identity so theres a inverse. Also it Associative thus it is a Subgroup of A_4. Also to point out, any two members in H be "multiplied" stay in H.

1c. $H = \left\{ \begin{pmatrix} 1&2&3&4 \\ 1&2&3&4 \end{pmatrix}, \begin{pmatrix} 1&2&3&4 \\ 2&3&1&4 \end{pmatrix}, \begin{pmatrix} 1&2&3&4 \\ 3&1&2&4 \end{pmatrix} \right\}$

Notice eH is going to be a left coset
Now pick an element of A_4 but not in H.

Pick $a_1 = \begin{pmatrix} 1&2&3&4 \\ 1&3&4&2 \end{pmatrix}$, $\begin{pmatrix} 1&2&3&4 \\ 1&3&4&2 \end{pmatrix} * H = \begin{pmatrix} 1&2&3&4 \\ 1&3&4&2 \end{pmatrix}, \begin{pmatrix} 1&2&3&4 \\ 2&1&4&3 \end{pmatrix}, \begin{pmatrix} 1&2&3&4 \\ 3&2&4&1 \end{pmatrix}$

Pick $a_2 = \begin{pmatrix} 1&2&3&4 \\ 1&4&2&3 \end{pmatrix}$, $a_2 * H = \begin{pmatrix} 1&2&3&4 \\ 1&4&2&3 \end{pmatrix}, \begin{pmatrix} 1&2&3&4 \\ 2&4&3&1 \end{pmatrix}, \begin{pmatrix} 1&2&3&4 \\ 3&4&1&2 \end{pmatrix}$

Pick $a_3 = \begin{pmatrix} 1&2&3&4 \\ 4&1&3&2 \end{pmatrix}$, $a_3 * H = \begin{pmatrix} 1&2&3&4 \\ 4&1&3&2 \end{pmatrix}, \begin{pmatrix} 1&2&3&4 \\ 4&2&1&3 \end{pmatrix}, \begin{pmatrix} 1&2&3&4 \\ 4&3&2&1 \end{pmatrix}$

4 left cosets
lemma, for any members a and b, $a * H = b * H$ or $a * H$ and $b * H$ have nothing in common,
Clearly they have nothing in common in this example.
Secondly, $|a * H| = |b * H|$ so there are the Same number of elements.

1d. Since $A_4 = \begin{pmatrix} 1&2&3&4 \\ 1&2&3&4 \end{pmatrix} H \cup \begin{pmatrix} 1&2&3&4 \\ 1&3&4&2 \end{pmatrix} H \cup \begin{pmatrix} 1&2&3&4 \\ 1&4&2&3 \end{pmatrix} H \cup \begin{pmatrix} 1&2&3&4 \\ 4&1&3&2 \end{pmatrix} H$

and each coset has the Same amount of element and nothing in common, Then $|A_4|/|H|$ has to be an integer.

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2. Given any group G with abstract multiplication denoted by $*$, and Identy element e, and a subgroup H, The left coset is $aH = \{a*h_1, a*h_2, ...\}$ Where $a \in G \setminus H$.
Then $G = e*H \cup a_1*H \cup a_2*H \cup ... \cup a_k*H$.
Also notice after picking your $a_1 \in G \setminus H$ then if there are other cosets then you have to make $a_2 \in G \setminus H \wedge a_1*H$, so if there were an a_3 then pick $a_3 \in G \setminus H \wedge a_1*H \wedge a_2*H$.
From this we get two lemmas. First for any member a and b, either $a*H = b*H$ or $a*H$ has nothing to do with $b*H$. Suppose $a*H$ has a common member with $b*H$. Then there exist h_1 in H and h_2 in H such that $a*h_1 = b*h_2$ multiply LHS by a^{-1} to get $h_1 = a^{-1}*b*h_2$ and multipy RHS by h_2^{-1} to get $h_1*h_2^{-1} = a^{-1}*b$, This implies that $a^{-1}*b$ belongs to H so a belongs to the coset $b*H$ therefore $a*H = b*H$. By how we constructed the coset, $|a*H| = |b*H|$. Using the left-coset decomposition and the fact that there are the same number of elements in each coset and each coset no intersection then $\frac{|G|}{|H|}$ must be an integer which is Langrange's theorem