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(1.)

{1,2,3,4}

Even permutations: $\begin{bmatrix}1&2&3&4\\1&2&3&4\end{bmatrix}$, $\begin{bmatrix}1&2&3&4\\4&3&4&2\end{bmatrix}$, $\begin{bmatrix}1&2&3&4\\1&4&2&3\end{bmatrix}$, $\begin{bmatrix}1&2&3&4\\2&3&1&4\end{bmatrix}$, $\begin{bmatrix}1&2&3&4\\2&4&3&1\end{bmatrix}$, $\begin{bmatrix}1&2&3&4\\3&1&2&4\end{bmatrix}$

$\begin{bmatrix}1&2&3&4\\3&2&4&1\end{bmatrix}$, $\begin{bmatrix}1&2&3&4\\4&1&3&2\end{bmatrix}$, $\begin{bmatrix}1&2&3&4\\4&2&1&3\end{bmatrix}$, $\begin{bmatrix}1&2&3&4\\1&3&4&2\end{bmatrix}$, $\begin{bmatrix}1&2&3&4\\1&4&2&3\end{bmatrix}$, $\begin{bmatrix}1&2&3&4\\4&3&2&1\end{bmatrix}$

A_4 has 12 members. To check that A_4 is a subgroup of S_4, we first start by noting that 12 divides 24. A_4 is the set of even permutations, with 12 elements, the odd set has 12 elements. Comparing the two partitions, A_4 is a subgroup of S_4.

(b) $\left(\begin{bmatrix}1&2&3&4\\1&2&3&4\end{bmatrix}, \begin{bmatrix}1&2&3&4\\2&3&1&4\end{bmatrix}, \begin{bmatrix}1&2&3&4\\3&1&2&4\end{bmatrix}\right)$

Proof:

Lagrange, if H is a subgroup of G, then $|G| = [G:H] \cdot |H|$

Thus, our left cosets are: IH, $(12)(34)H$, $(13)(24)H$

Since our statement holds, $[G:H]$ is the value of our subgroup H. So H is a subgroup of A_4.

(c.) The left cosets of H in A_4 are $(12)(34)(I)$, $(12)(34)(123)(4)$, $(12)(34)(132)(4)$
$(13)(24)(I)$, $(13)(24)(123)(4)$, $(13)(24)(132)(4)$, $(14)(23)(I)$, $(14)(23)(123)(4)$, $(14)(23)(132)(4)$

There are 8 left cosets. They all have the same number of elements because the left cosets are an equivalence relation on A_4.

(d) This illustrates the Lagrange Theorem because for any finite group G and its subgroup H $|G| \setminus |H|$ is an integer. We see this because it holds the same value as $[G:H] \cdot |H|$
$[G] = 24 = [G:H] \cdot |H| = 8 \cdot 3 = 24$.

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(2.) left coset of g is $gH := \{g \cdot h : h \in H\}$

Right coset of g is $Hg = \{h \cdot g : h \in H\}$

They have the same number of elements because multiplication is equivalent/commutative.
They can never have common members, and they have the same number of elements.

So Lagrange's theorem holds true.