hw 16

a. $\begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\1&3&4&2\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\1&4&2&3\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\2&1&4&3\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\2&3&1&4\end{pmatrix}$,
$\begin{pmatrix}1&2&3&4\\2&4&3&1\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\3&1&2&4\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\3&2&4&1\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\3&4&1&2\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\4&1&3&2\end{pmatrix}$,
$\begin{pmatrix}1&2&3&4\\4&2&1&3\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\4&3&2&1\end{pmatrix}$

12 members

Each of these are an even permutation, and multiplying And an even permutation yields an even by a even permutation yields an even permutation.

b. 1. contains identity element ✓

2. $\begin{pmatrix}1&2&3&4\\2&3&1&4\end{pmatrix}\begin{pmatrix}1&2&3&4\\3&1&2&4\end{pmatrix}=\begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix}$ so nonidentity

elements are inverses and so all products of elements stays within the group

c. $a1=\begin{pmatrix}1&2&3&4\\1&3&2&4\end{pmatrix}$

→ $a1\times H=\left\{\begin{pmatrix}1&2&3&4\\1&3&2&4\end{pmatrix},\begin{pmatrix}1&2&3&4\\2&1&3&4\end{pmatrix},\begin{pmatrix}1&2&3&4\\3&2&1&4\end{pmatrix}\right\}$

$a2=\begin{pmatrix}1&2&3&4\\1&4&2&3\end{pmatrix}$

→ $a2\times H=\left\{\begin{pmatrix}1&2&3&4\\1&4&2&3\end{pmatrix},\begin{pmatrix}1&2&3&4\\2&4&3&1\end{pmatrix},\begin{pmatrix}1&2&3&4\\3&4&1&2\end{pmatrix}\right\}$

$a3=\begin{pmatrix}1&2&3&4\\3&2&4&1\end{pmatrix}$

→ $a3\times H=\left\{\begin{pmatrix}1&2&3&4\\3&2&4&1\end{pmatrix},\begin{pmatrix}1&2&3&4\\1&3&4&2\end{pmatrix},\begin{pmatrix}1&2&3&4\\2&1&4&3\end{pmatrix}\right\}$

3 left cosets, all of which have the same number of elements as they're all a product of a permutation of size 4 with a subgroup of size 3 with permutations of size 4. They can't have a common element because they are disjoint as ?.

d. I don't understand Lagrange's theorem.

2. If H is a subgroup of G, then [illegible] $|G| = [G:H] \cdot |H|$

I do understand this version even less.