

# HW 16

OK to post

$$1) a) \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$$

$A^4$  has 2 subgroups. For any members  $a$  &  $b$   
either  $a^*H = b^*H$  has nothing  
in common with  $b^*H$

b)  $H$  contains the identity element and  
we see that  $(A^4)^2$  gets us back to  
the  $A^2$ . And  $A^2 * A^4$  gets us the identity  
permutation

c) confused

d) we explicitly construct left-coset decomposition  
of  $A^4$  into cosets of  $H$ , each having  
the same number of elements. And none  
of them overlapping.

2) Suppose that  $a^*H$  has a common member with  $b^*H$   
There exists  $h_1$  in  $H$  and  $h_2$  in  $H$  such  
that  $a^*h_1 = b^*h_2$ . Now multiply from the right by  $h_2^{(-1)}$   
 $h_1^*h_2^{(-1)} = a^{(-1)} * b$   
Hence  $a^{(-1)} * b$  belongs to  $H$  hence  $a$  belongs to the coset  $b^*H$   
Hence  $a^*H = b^*H$   $|G|/|H|$  is an int