Homework 16

1. a. $\begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\1&3&4&2\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\1&4&2&3\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\1&4&2&3\end{pmatrix}$,

$\begin{pmatrix}1&2&3&4\\2&1&4&3\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\2&3&1&4\end{pmatrix}$, $\begin{pmatrix}1&2&3&4\\2&4&3&1\end{pmatrix}$

$\begin{pmatrix}1&2&3&4\\3&1&2&4\end{pmatrix}$ $\begin{pmatrix}1&2&3&4\\3&2&4&1\end{pmatrix}$ $\begin{pmatrix}1&2&3&4\\3&4&1&2\end{pmatrix}$ $\begin{pmatrix}1&2&3&4\\4&1&3&2\end{pmatrix}$

$\begin{pmatrix}1&2&3&4\\4&2&1&3\end{pmatrix}$ $\begin{pmatrix}1&2&3&4\\4&3&2&1\end{pmatrix}$

A_4: 12 members.

\# of inversion of the symmetric group is the same, thus, A_4 is a subgroup of $\{1, 2, 3, 4\}$

b. $H = \left\{\begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix}, \begin{pmatrix}1&2&3&4\\2&3&1&4\end{pmatrix}, \begin{pmatrix}1&2&3&4\\3&1&2&4\end{pmatrix}\right\}$

Because, there has $\begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix}$

Every element multiplied will be the same as the group

Every element in the group belongs to the H

c. Coset in G and the cosets are $\Rightarrow 4$

(i) $I(H) = H$

(ii) $(2\ 3\ 4)H = \{(2, 3, 4), (1 4 2), (13)(24)\}$

(iii) $(13)(24)H = \{(13)(24), (2, 34), (142)\}$

(iv) $(14)(23)H = \{(14)(23)\ (124), (134)\}$

There are four distinct left cosets of H in A_4. But total number of cosets of H is $A_4 = 12$.

· Each one of them has same number of element because we are applying the elements of group on same subgroup of H.

· Since intersection of all cosets of H in G is empty set

d. $H = 3$ and A_4 has order 12 Also 3 divide 12

i.e order of subgroup divides order of finite

So this is example illustrates the famous group Lagrange theorem that for any finite group G and any of its subgroup H, $|G|/|H|$ is always an integer,

Q. Let G be a finite group

Let H be a subgroup of G

Consider a Congruence relation for $a, b \in G$ $a \equiv b \mod H$ iff $ab^{-1} \in$

We know Congruence relation = Equivalence relation

$\forall\, a \in G$ $[a] = Ha \rightarrow$ Right coset

Suppose $[a_1], [a_2], \ldots [a_k]$ are distinct equivalence classes of G

$$G = [a_1] \cup [a_2] \ldots \cup [a_k]$$

$$G = Ha_1 \cup Ha_2 \ldots \cup Ha_k$$

$$o(h) = o(Ha_1) + o(Ha_2) + \cdots o(Ha_k)$$

$$[o(Ha)] = o(H)$$

$$o(H) = \underbrace{oH + o(H) + \cdots + o(H)}_{k\text{-times}}$$

$$o(H) = k(o(H))$$

$$\frac{o(h)}{o(H)} = k \Rightarrow o(H) \text{ divide } o(G)$$