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It is OK to post the homework in your web-site

1. $(1,2,3)$, number of inversions: 0
$(1,3,2)$, number of inversions: 1
$(2,1,3)$, number of inversions: 1
$(2,3,1)$, number of inversions: 2
$(3,2,1)$, number of inversions: 3
$(3,1,2)$, number of inversions: 2

2. Set of inversions $= \{52, 53, 54, 74, 76\}$
Number: 5

3. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & B \end{pmatrix}$

In every legal move: the invariant (number of inversions + taxicab distance of B from the top-right concer) always changes by either 2 or 0.
Then number of inversions that is changed in one legal move is always odd (often 1).
Taxicab distance of B = 16 always changes by ± 1.
In every legal move the above invariant changes by an even, i.e. have the same parity.
Hence no matter how many legal moves, the parity of the invariant is the same.

4. A group consists a set of objects, $\{g_1, g_2, \cdots, g_m\}$
An operation called "multiplication" denoted by $*$ such that
(i) For any members g, g' of G, $g*g'$ is also in G

(ii) There exists a special member, usualy called e (the idenity member) with the property $g*e=g$ for every number of G.

(iii) For any three numbers of G, g_1, g_2, g_3 : $(g_1*g_2)*g_3 = g_1*(g_2*g_3)$ associativity axiom.

(iv) Every member g of G has a so-called inverse, denoted by $g^{\{-1\}}$ with the property $g*g^{\{-1\}}=e$; $g^{\{-1\}}*g=e$