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Homework 15

(1) For each of the 6 permutations of  $\{1, 2, 3\}$  find the number of inversions.

$$\{1, 2, 3\} \rightarrow 0$$

$$\{1, 3, 2\} \rightarrow 1$$

$$\{2, 1, 3\} \rightarrow 1$$

$$\{2, 3, 1\} \rightarrow 2$$

$$\{3, 2, 1\} \rightarrow 2$$

$$\{3, 1, 2\} \rightarrow 3$$

$$\begin{aligned} (2) \quad & (1\ 5\ 2\ 3\ 7\ 4\ 6) \downarrow +1 \\ & (1\ 2\ 5\ 3\ 7\ 4\ 6) \downarrow +2 \\ & (1\ 2\ 3\ 5\ 7\ 4\ 6) \downarrow +3 \\ & (1\ 2\ 3\ 5\ 6\ 4\ 7) \downarrow +4 \\ & (1\ 2\ 3\ 6\ 5\ 4\ 7) \downarrow +5 \\ & (1\ 2\ 3\ 4\ 5\ 6\ 7) \downarrow +5 \end{aligned}$$

5 inversions

(4) A group has closure  $a \oplus b \in S$   
 associativity  $(a \oplus b) \oplus c = a \oplus (b \oplus c)$ .  
 identity  $a \oplus I = I \oplus a = a$   
 and an inverse  $a \oplus a^{-1} = a^{-1} \oplus a = I$ .

(5) We know matrix multiplication is associative

$$\text{The } I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The inverse of any  $2 \times 2$  matrix if  $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$  would be  $\frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$