Sarah Cafaric | Homework 15 | 11/7/2021

1. $\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}$ → number of inversions: 0
$\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}$ → number of inversions: 1
$\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}$ → number of inversions: 3
$\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}$ → number of inversions: 1
$\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}$ → number of inversions: 1
$\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}$ → number of inversions: 2

2. (1523746) - inversions: (52, 53, 54, 74, 76)
number of inversions: 6

3. It is not possible because the taxicab distance stays the same, but the number of inversions is 1. However, with each move, the invariant (taxicab distance + # of inversions) changes by an even number. So, it is not possible to have a legal move which leads to the image.

4. A group is a set of objects that have a property of "multiplication" such that for any elements f and f * g is an element of the group.
There is an identity element, denoted e, such that for any element f $f * e = f$ and $e * f = f$.
All elements have an inverse denoted f^{-1} such that
$f * f^{-1} = e$ and $f^{-1} * f = e$
There is associativity for any elements f, g, h
$f * (g * h) = (f * g) * h$

5. (i) The identity element is $\begin{bmatrix}1&0\\0&1\end{bmatrix}$
(ii) To find the inverse of matrix $A = \begin{bmatrix}a&b\\c&d\end{bmatrix}$

$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix}d&-b\\-c&a\end{bmatrix}$, and it is a part of the group because $\det A^{-1} = 1(da-bc) = 1$