Quin Buob

1) {1,2,3} {1,3,2} {2 1 3} {2 3 1} {3 2 1} {3,1,2}

Inversions: 0 1 1 2 3 2

3) This is impossible because the invariant changes its parity.

Invariant = sum of inversions of each row + Taxi cab distance of the blank from top right

For every move the sum of the inversions changes by an odd number,
For every move the taxicab distance changes by ± 1

From this the Invariant changes by an even number so the parity of the invariant must remain the same for all legal moves

For the initial problem:

Inversions = 0, Taxicab dist = 6 => Invariant = 6 => Even

For the wanted solution

Inversions = 1, taxicab dist = 6 => Invariant = 7 => Odd

There fore this is not a possible solution to the fifteen puzzle because the solution's Invariant is odd, which means that it cannot be obtained by legal moves.

2) {1 5 2 3 7 4 6} Inversions = 3 + 2 = 5

4) A group, G, is a set that consists of elements; $\{g_1 \dots g_n\}$, which follows a set of rules when undergoing a given transformation, $*$

i) For any members g and g' of G, then $g*g'$ is also a member of G

ii) There also exists an element, e, in group G s.t.

$$e*g=g \text{ and } g*e=g \text{ for every member of } G$$

iii) For any 3 member of G; g_1, g_2, g_3:

$$(g_1 * g_2) * g_3 = g_1 * (g_2 * g_3)$$

iv) Every member of G has an inverse element in G, g^{-1}, s.t.

$$g*g^{-1}=e \text{ and } g^{-1}*g=e$$

5) ~~All~~ The set of all 2x2 matrces with determinant 1 because :

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$ For this to be a member of this group $(ae+bg)(cf+dh) - (af+bh)(ce+dg) = 1$

We know :

$ad-bc=1$ and $eh-fg=1$ then we know this to be true from linear algebra

i) the identity element is $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\det \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1$ and

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a+0 & b+0 \\ c+0 & d+0 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

ii) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = A$ and $\det A = ad-bc = 1$

$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ * $\det A^{-1} = \frac{da-bc}{ad-bc} = 1$

$AA^{-1} = I$

6) I don't know

7) I don't know