HOMEWORK 15 NINA CHALGERI

①

{1, 2, 3} 0 inversions ~~permutations~~

{1, 3, 2} 1 inversions (2, 1)

{2, 1, 3} 1 inversion (1, 2)

{2, 3, 1} 2 inversions (3, 1) (2, 3)

{3, 1, 2} 2 inversions (1, 2) (1, 3)

{3, 2, 1} 3 inversions (3, 2) (3, 1) (2, 1)

②

1 5 2 3 7 4 6

↓ ↓ ↓ ↓ ↓ ↓ ↓

0 + 3 + 0 + 0 + 2 + 0 + 0 = 5 total inversions

③ * It is impossible to solve the 15-puzzel with legal moves.

PROOF.

· an invariant is the number of inversions plus the taxicab distance of the blank space from the top-~~right~~ left corner : $(i+j) - 2$

* lemma: in every legal move, this invariant always changes by either 2 or 0.
  ↳ the number of inversions that is changed in one legal move is always odd (often 1)

* Lemma: In every legal move, the above variant changes by an even number ⇒ (have the same parity).

⇒ Hence, no matter how many legal moves, the parity of the invariant is the same.

∴ ↳ This means that the 15-puzzle ~~that~~ ~~is~~ only has one inversion between the start and the end goal; ~~both~~ and the taxicab distance is 6. 1 is odd and 6 is even so ~~they are in a~~ ~~parity~~ these two numbers have different parities.

→ Impossible.

④ * Abstract finite group.

- A group, G, of order m consists of a set of objects $\{g_1, g_2, \cdots, g_m\}$
- an operation called multiplication denoted by $*$ such that:

(i) for any numbers g, g' of G : $g * g' \in G$

(ii) there exists a special number usually called e (identity number) with properties $g * e = e * g = g$ for every element in G

(iii) For any 3 numbers of G: g_1, g_2, g_3

$(g_1 * g_2) * g_3 = g_1 * (g_2 * g_3) \Rightarrow$ Associativity

(iv) every number g ~~does~~ of G has so-called inverse denoted by g^{-1} with $g * g^{-1} = e$ and $g^{-1} * g = e$

⑤ I DONT KNOW

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