

## Homework 15

- ① {1, 2, 33}: No inversions  
{1, 3, 23}: (23) - 1  
{2, 1, 33}: (21) - 1  
{2, 3, 13}: (21, 31) - 2  
{3, 2, 1}: (32, 31, 21) - 3  
{3, 1, 23}: (31, 32) - 2

② 1523746:

$$(52, 53, 54, 74, 76) - 5$$

- ③ Let's call the blank space 16. Look at the number of inversions of the permutation then  $S = \text{"parity of the \# inversions + row \# + col \#"} \pmod 2$ . In any permutation, exchanging any 2 entries, the diff between  $\#$  of inversions is always odd. Since any of the legal moves preserves the parity of  $S$ , no matter how many moves you make, if it starts out even it remains even (as shown in class). Hence, it is impossible to have  $S$  change from even to odd or odd to even.

- ④ A group consists of a set of objects  $\{g_1, g_2, \dots, g_n\}$  (this is a group of order  $n$ ). An operation called "multiplication" denoted by  $*$  such that for  $g$  and  $g'$  in  $G$ ,  $g * g'$  is also in  $G$ . There exists a special member, usually called  $e$  (The identity number) with the property  $g * e = g$  and  $e * g = g$  for every number of  $G$ . For any three members of  $G$ ,  $(g_1 * g_2) * g_3 = g_1 * (g_2 * g_3)$ . Every member of  $G$  has an inverse denoted by  $g^{-1}$  with  $g * g^{-1} = e$ ,  $g^{-1} * g = e$ .

⑤ Sorry I'm not sure. I will re-visit lecture before the quiz

⑥ Same as 5