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1. Permutation | inversion

Permutation	inversion
{1, 2, 3}	none
{2, 3, 1}	{31}
{3, 1, 2}	{32, 31}
{3, 2, 1}	{32, 31, 21}
{2, 1, 3}	{21}
{1, 3, 2}	{32}

3 inversions

2. 1523746

inversion = {52, 53, 54, 74, 76}

5 inversions.

Scanned with CamScanner

3. If you start with $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 13 \end{pmatrix}$

it is impossible to get to $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 15 & 14 & 13 \end{pmatrix}$

Since every one legal move the inversion is odd and taxi cab distance does not change then inversion + taxi cab = odd number = invariant but invariant changes by an even number So the invariant is an even number not an odd number thus a contradiction So it is impossible to get to the second matrix.

4. A group consist of a set ob objects $\{g_1, g_2, \ldots g_n\}$ and an operation called "multiplication" such that $g * g'$ is also in G. There also exist a member called e such that $g * e = g$ and $e * g = g$. Also there is associativity and inverses.

5. Suppose we have a matrix 2×2 $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $a, b, c, d \in \mathbb{Z}$

and $\frac{1}{ad-bc} = 1$ then the identity

element would be $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Since $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

The inverse would be a matrix that if you multiply it by $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ you get back the identity matrix

So $\left(\frac{1}{ad-bc}\right)\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ would be the inverse matrix

Since $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

And associativity is known so the 2×2 matrix with a determinant 1 is a group.

6. Suppose we have a matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

where a, b, c, d can be the entries $\{0, 1, 2\}$

with $\frac{1}{ad-bc} = e$ where $e \neq 0$.

Also the operation is matrix multiplication mod 3.
First the identity matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ wouldn't change since $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ no matter what the inputs are.
Secondly, The inverse would be
$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where this time $\frac{1}{ad-bc}$ is not

1 but e where $e \neq 0$ this gives us

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

And we know its associative so it is a finite group.