1)

1	2	3
1	3	2
2	1	3
2	3	1
3	2	1
3	1	2

All 6 permutations of {1,2,3}

The inversion for each is:

Permutations			Inversions
1	2	3	0
1	3	2	1
2	1	3	1
2	3	1	1
3	2	1	3
3	1	2	2

2)

1	5	2	3	7	4	6	Inversions
0	3	0	0	2	0	0	5

3)

In this case, the n x n matrix, n=4 which is even. Now counting where the blank is within the puzzle, makes this on the 4th row, however we need to count from the bottom making this the first row, being an odd number. With this, having an odd inversion as 15 preceded by 14 is the only inversion, this makes the puzzle impossible to solve as the blank space must be at an even row number.

4)

A group is a set that uses an operation combining two elements to create a third while maintaining an identity and inverse element.

5)

To prove that such a 2x2 matrix using multiplication exists with a determinant of 1 is a group. For an identity element, we say that the matrix A, for all values of S, one of these elements has id*A=A*id=A. For that the determinant of A is the same as the determinant of A*id, and thus det(A)*det(id), goes to det(id)=1 makes an identity of a matrix exist. For the inverse, we conduct it similarly with A*A^-1=id, thus the determinant of the above separates to det(A)=det(A^-1)=det(i)=1. This makes the group a finite group

6)

For the closure axiom, with numbers [0 1 2], in a 2x2 matrix, these two elements must have a nonzero determinant where both elements should have the same determinant to achieve closure. Using the following matrix for a and b respectively:

A

1	0
0	2

B

2	0
0	1

Using these as examples, the determinant of A is 2, and B is 2.

C

2	0
0	2

With 2*2=det(c)is true as det(c)=4 thus, closure.

Associativity of the matrix is known from linear algebra.

Now for an identity matrix, having A*id=A, as with the identity matrix, and the config of our matrix, this allows this to occur, thus the det(A*id)=det(A)det(id), thus det(id)=1

And now for the inverse of the group. Where the multiplication of A^-1*A = id matrix.
A^-1

1	0
0	1/2

Thus the multiplication of both is the identity matrix. Thus this forms a group. This is also a finite group. The amount of elements that can occur is 3^4=81 possible elements