hw 15

1. $\{1,2,3\}: 0$
$\{1,3,2\}: 1$
$\{2,3,1\}: 2$
$\{2,1,3\}: 1$
$\{3,1,2\}: 2$
$\{3,2,1\}: 2$

2. 3

3. I read the participation solution but I dont understand this question still.

4. A set of objects such that the product of any two objects is also in the group.

5. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $B = \begin{bmatrix} j & k \\ l & m \end{bmatrix}$ be two matrices with integer entries and determinants 1.

Then $ad - bc = 1$ and
$jm - kl = 1$.

Then $A \times B = \begin{bmatrix} aj + bl & ak + bm \\ cj + dl & ck + dm \end{bmatrix}$

The determinant of this is
$(aj + bl)(ck + dm) - (cj + dl)(ak + bm)$
$= \cancel{ajck} + ajdm + blck + \cancel{bldm} - \cancel{ajck} - bcjm - adlk - \cancel{bdlm}$
$= adjm + bckl - bcjm - adkl$
$jm(ad - bc) - kl(ad - bc)$
$= jm - kl$
$= 1$

Because each term of $A \times B$ is the sum of products of integers, each term is also an integer.
Since the determinant is also 1, $A \times B$ is in the set so the defined set is a group. It isn't a finite group as there are infinitely many integers.

i) the identity element for $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

ii) Inverse of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Suppose $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} j & k \\ l & m \end{bmatrix}$

are in the defined set.

Then ~~Aft~~ $ad - bc \neq 0$ and $jm - kl \neq 0$ and

$ad - bc \leq 4$ and $jm - kl \leq 4$.

As determined earlier the determinant of $A \times B$ is

$adjm + bckl$.

(I'm not sure how to prove)..

this is a finite group with $3^4 = 81$ elements.