Homework 15

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Problem 1

1. (1,2,3) → # of inversions is 0

2. (1,3,2) → # of inversions is 1

3. (2,1,3) → # of inversions is 1

4. (2,3,1) → # of inversions is 2

5. (3,1,2) → # of inversions is 2

6. (3,2,1) → # of inversions is 3

Problem 2

# of inversions in 1523746

List of inversions: (5 2), (5 3), (5 4), (7 4), (7 6)

Number of inversions is 5

Problem 3

We define the invariant as the number of inversions plus the taxicab distance of the blank spot. In any one legal move, the invariant will change only by either 2 or 0. As such, it is impossible to move from a position of odd invariance to a position of even invariance or vice versa. Since the scenario posed is such a scenario, it is therefore impossible to go from one position to the other using legal moves.

Problem 4

A group is a set G which, under a given operation:

i) satisfies the properties of associativity
ii) has an identity element
iii) has a unique inverse element b for every element a within G

Problem 5

Associativity is known from linear algebra. The identity element I of the group is the identity matrix $\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$ which of course satisfies the equation that for any element a, a*I and I*a both must equal a. We know that a given square matrix is invertible if and only if its determinant is non-zero. As such for every matrix in this group (which all have determinant 1) we can say that there is an inverse matrix for each and every element in this group. We can find the inverse of a given matrix A by finding the matrix B that satisfies the equation $AB=BA=I_n$. This group is decidedly not a finite group since there are infinite numbers of matrices with determinant 1.