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Homework 15 11/07/21

1. (1 2 3) - 0 inversions
(1 3 2) - 1 inversion: (32)
(2 1 3) - 1 inversion: (21)
(2 3 1) - 2 inversions: (21, 31)
(3 1 2) - 2 inversions: (31, 32)
(3 2 1) - 3 inversions: (32, 31, 21).

2. 1 5 2 3 7 4 6
set of inversions: {52, 53, 54, 74, 76}
Number of inversions: 5

3. It is impossible to start with

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & \end{pmatrix} \text{ and get } \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 15 & 14 & \end{pmatrix}$$

The square on the left has 0 inversions, but the square on the right has 1. Thus, we would have to make a set of moves that changes the number of invesions by net 1.
By the Lemma from class any move is going to change the number of inversions by an even number. Therefore, it is impossible to make a square with 1 inversion from a square of 0 inversions.

4. Group: "a set equipped with an operation that combines any two elements to form a third element while being associative as well as having an identity and inverse elements." (wiki article on Groups).

5. Take $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where $a, b, c, d \in \mathbb{Z}$ and $ad - bc = 1$.
   1. Associativity is known from linear algebra.
   Since $\det = 1 \neq 0$, the matrix has:
      i) an identity element:
      $$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
      ii) and an inverse:
      $$\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

6. Take $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where $a, b, c, d \in \{0, 1, 2\}$ and $ad - bc \neq 0$.
   Then:
   1. Associativity is known from linear algebra
   2. Identity element: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$] since $\det \neq 0$.
   3. Inverse: $\frac{1}{ad-bc}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

Therefore it is a finite group.