

HW 15

1) $(1\ 2\ 3)$ $(1\ 3\ 2)$ $(2\ 3\ 1)$ $(2\ 1\ 3)$ $(3\ 2\ 1)$ $(3\ 1\ 2)$
↓ ↓ ↓ ↓ ↓ ↓
(none) $(3\ 2)$ $(2\ 1)$ $(3\ 1)$ $(2\ 1)$ $(3\ 2)$ $(2\ 1)$ $(3\ 1)$ $(3\ 2)$
inversions ⇒ 0 1 2 1 2 2

2) $1\ 5\ 2\ 3\ 7\ 4\ 6$
 $(5\ 2)$ $(5\ 3)$ $(5\ 4)$ $(7\ 4)$ $(7\ 6)$ → # inversions 5

3) We know that the lemma: - In any perm., if we exchange any 2 entries, the difference between their number of inversions is always odd.

In a 15-puzzle, there are 2 types of legal moves:

moving horizontally & moving vertically

we know the parity remains the same for the 2.

It is impossible to go from the initial state to the state where 14 & 15 are changed since the parity is odd.

4) Group: a set equipped with an operation that combines any 2 elements to form a third element while being associative as well as having an identity element & inverse elements.

5) 2×2 matrix is an open subset of the space of all $n \times n$ matrices, because it is given by the inequality: $\det(A) \neq 0$, where A denotes the 2×2 matrix.

The integer b in $a \cdot b$ is the inverse element

6) Unsure. I am stuck.