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Homework for Lecture 14 - OK to post

① a.) 1 goes to 4, 4 goes to 7
2 goes to 5, 5 goes to 1
3 goes to 7, 7 goes to 2
4 goes to 6, 6 goes to 6
5 goes to 8, 8 goes to 9
6 goes to 1, 1 goes to 5
7 goes to 9, 9 goes to 3
8 goes to 3, 3 goes to 8
9 goes to 2, 2 goes to 4

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 1 & 2 & 6 & 9 & 5 & 3 & 8 & 4 \end{pmatrix}$$

b.) 1 goes to 5, 5 goes to 8
2 goes to 7, 7 goes to 6
3 goes to 4, 4 goes to 1
4 goes to 1, 1 goes to 5
5 goes to 8, 8 goes to 3
6 goes to 6, 6 goes to 2
7 goes to 2, 2 goes to 7
8 goes to 3, 3 goes to 4
9 goes to 9, 9 goes to 9

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 6 & 1 & 5 & 3 & 2 & 7 & 4 & 9 \end{pmatrix}$$

② 1 goes to 3, 3 goes to 5
2 goes to 4, 4 goes to 2
3 goes to 5, 5 goes to 1
4 goes to 2, 2 goes to 4
5 goes to 1, 1 goes to 3

$$\pi^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 2 & 1 & 4 & 3 \end{pmatrix}$$

1 goes to 5, 5 goes to 1
2 goes to 2, 2 goes to 4
3 goes to 1, 1 goes to 3
4 goes to 4, 4 goes to 2
5 goes to 3, 3 goes to 5

$\pi^3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 3 & 2 & 5 \end{pmatrix}$

1 goes to 1, 1 goes to 3
2 goes to 4, 4 goes to 2
3 goes to 3, 3 goes to 5
4 goes to 2, 2 goes to 4
5 goes to 5, 5 goes to 1

$\pi^4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 5 & 4 & 1 \end{pmatrix}$

1 goes to 3, 3 goes to 5
2 goes to 2, 2 goes to 4
3 goes to 5, 5 goes to 1
4 goes to 4, 4 goes to 2
5 goes to 1, 1 goes to 3

$\pi^5 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 1 & 2 & 3 \end{pmatrix}$

1 goes to 5, 5 goes to 1
2 goes to 4, 4 goes to 2
3 goes to 1, 1 goes to 3
4 goes to 2, 2 goes to 4
5 goes to 3, 3 goes to 5

$\pi^6 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$

It took 6 powers to get to the identity permutation.

3. The cycles are $\begin{pmatrix} 1 & 4 & 6 \\ 4 & 6 & 1 \end{pmatrix} \begin{pmatrix} 2 & 5 & 8 & 3 & 7 & 9 \\ 5 & 8 & 3 & 7 & 9 & 2 \end{pmatrix}$

which is $(1\ 4\ 6)(2\ 5\ 8\ 3\ 7\ 9)$ in cycle notation. The lengths of the cycles are 3 and 6, so the $\text{lcm}(3, 6) = 6$, which is the smallest i such that π^i is the identity permutation.

④ The cycles are $\begin{pmatrix} 1 & 9 \\ 9 & 1 \end{pmatrix}\begin{pmatrix} 2 & 4 & 7 \\ 4 & 7 & 2 \end{pmatrix}\begin{pmatrix} 3 & 5 & 6 & 8 & 10 \\ 5 & 6 & 8 & 10 & 3 \end{pmatrix}$

which is $(1\ 9)(2\ 4\ 7)(3\ 5\ 6\ 8\ 10)$ in cycle notation. The lengths of the cycles are 2, 3, and 5, so the $\mathrm{lcm}(2,3,5) = 30$, which is the smallest i such that π^i is the identity permutation.

⑤ Step 1: Flip the two rows

$$\begin{pmatrix} 4 & 9 & 5 & 8 & 6 & 7 & 2 & 1 & 10 & 3 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{pmatrix}$$

Step 2: Reorganize them so top row follows convention of 1-10

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 8 & 7 & 10 & 1 & 3 & 5 & 6 & 4 & 2 & 9 \end{pmatrix}$$