Hw14

Wednesday, November 3, 2021 2:22 AM

1. Perform the following permutation-products

a.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 5 & 7 & 6 & 8 & 1 & 9 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 4 & 8 & 7 & 1 & 6 & 2 & 9 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 1 & 2 & 6 & 9 & 5 & 3 & 8 & 4 \end{pmatrix}$$

b.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 7 & 4 & 1 & 8 & 6 & 2 & 3 & 9 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 7 & 4 & 1 & 8 & 2 & 6 & 3 & 9 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 6 & 1 & 5 & 3 & 2 & 7 & 4 & 9 \end{pmatrix}$$

2. Let

$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 2 & 1 \end{pmatrix} ,$$

find $\pi, \pi^2, \ldots$ until you get the identity permutation. How many powers did you have to do?

$$\pi^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 2 & 1 & 4 & 3 \end{pmatrix} \qquad \pi^4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 5 & 4 & 1 \end{pmatrix}$$

$$\pi^3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 3 & 2 & 5 \end{pmatrix} \qquad \pi^6 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} \quad 6$$

3. Express the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 5 & 7 & 6 & 8 & 1 & 9 & 3 & 2 \end{pmatrix} ,$$

as a product of disjoint cycles. What is the smallest i such that π^i is the identity permutation?

$\pi = (146) \overset{\text{and}}{} (258379)$ $\quad i = 6$

$\pi = (1\,4\,6)$ and $(2\,5\,8\,3\,7\,9)$

$3 \qquad 6$

$\mathrm{lcm}(3,6) = 6$

$i = 6$

4. What is the smallest i such that π^i is the identiy permutation, if

$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 9 & 4 & 5 & 7 & 6 & 8 & 2 & 10 & 1 & 3 \end{pmatrix}$$

$\pi = (1\,9)\,(2\,4\,7)\,(3\,5\,6\,8\,10)$

$\mathrm{lcm}(2,3,6) = 6$

$i = 6$

5. Find π^{-1} if

$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 4 & 9 & 5 & 8 & 6 & 7 & 2 & 1 & 10 & 3 \end{pmatrix}$$

$\pi = (1\,4\,8)(2\,9\,10\,3\,5\,6\,7)$

$\pi^{-1} = (8\,4\,1)(7\,6\,5\,3\,10\,9\,2)$

$$\pi^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 8 & 7 & 10 & 1 & 3 & 5 & 6 & 4 & 2 & 9 \end{pmatrix}$$