HW 14

1a $p = \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 5 & 7 & 6 & 8 & 1 & 9 & 3 & 2 \end{matrix}$

$q = \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 4 & 8 & 7 & 1 & 6 & 2 & 9 & 3 \end{matrix}$

$$pq = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 6 & 3 & 9 & 4 & 1 & 5 & 2 & 7 \end{pmatrix}$$

$$pq = (1,8,2,6)(4,9,7,5)$$

1b. $p = \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 7 & 4 & 1 & 8 & 6 & 2 & 3 & 9 \end{matrix}$

$$q = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 7 & 4 & 1 & 8 & 2 & 6 & 3 & 9 \end{pmatrix}$$

$$pq = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 2 & 1 & 5 & 3 & 7 & 6 & 4 & 9 \end{pmatrix}$$

Problem 2

π_1 $\qquad\qquad\qquad$ π_2

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 2 & 1 \end{pmatrix}$$

$$\pi^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 2 & 1 & 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 2 & 1 \end{pmatrix}$$

$$\pi^3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 3 & 2 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 2 & 1 \end{pmatrix}$$

$$\pi^4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 5 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 2 & 1 \end{pmatrix}$$

$$\pi^5 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 2 & 1 \end{pmatrix}$$

$$\pi^6 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$

3. $(1,4,6)(2,5,8,3,7,9)$ is an expression of the permutation as a product of disjoint cycles
The smallest such i such that π^i is the identity permutation is $i=6$

$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 5 & 7 & 6 & 8 & 1 & 9 & 3 & 2 \end{pmatrix} \quad \pi^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & 8 & 9 & 1 & 3 & 4 & 2 & 7 & 5 \end{pmatrix}$$

$$\pi^3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 3 & 2 & 4 & 7 & 6 & 5 & 9 & 8 \end{pmatrix} \quad \pi^4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 7 & 5 & 6 & 9 & 1 & 8 & 2 & 3 \end{pmatrix}$$

$$\pi^5 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & 9 & 8 & 1 & 2 & 4 & 3 & 5 & 7 \end{pmatrix} \quad \pi^6 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{pmatrix}$$

4. $\pi = (1,9)(2,4,7)(3,5,6,8,10)$
LCM of cycle lengths 2, 3, and 5 is 30 so the smallest i that satisfies the condition is $i=30$

5. $$\pi^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 8 & 7 & 10 & 1 & 3 & 5 & 6 & 4 & 2 & 9 \end{pmatrix}$$