

Homework 14.

$$1a. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 5 & 7 & 6 & 8 & 1 & 9 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 4 & 8 & 7 & 1 & 6 & 2 & 9 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 1 & 2 & 6 & 9 & 5 & 3 & 8 & 4 \end{pmatrix}$$

$$b. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 7 & 4 & 1 & 8 & 6 & 2 & 3 & 9 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 7 & 4 & 1 & 8 & 2 & 6 & 3 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 6 & 1 & 5 & 3 & 2 & 7 & 4 & 9 \end{pmatrix}$$

$$\pi^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 2 & 1 & 4 & 3 \end{pmatrix} \quad \pi^6 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 2 & 1 & 4 & 3 \end{pmatrix}$$

$$\pi^3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 5 & 4 & 1 \end{pmatrix}$$

$$\pi^4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 2 & 1 & 4 & 3 \end{pmatrix}$$

π^5 is the identity permutation

$$3. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 5 & 7 & 6 & 8 & 1 & 9 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 6 & 7 & 8 & 9 \\ 9 & 8 & 7 & 6 \end{pmatrix}$$

Since the cycle lengths are 5 and 4, so the smallest i is 6

$$4. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 9 & 4 & 5 & 7 & 6 & 8 & 2 & 10 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 6 & 7 & 8 & 9 & 10 \\ 10 & 9 & 8 & 7 & 6 \end{pmatrix}$$

Since the cycle lengths are 5 and 5, so the smallest i is 5

$$5. \pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 4 & 9 & 5 & 8 & 6 & 7 & 2 & 1 & 10 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 4 & 9 & 5 & 8 & 6 & 7 & 2 & 1 & 10 & 3 \end{pmatrix}$$

$$\pi^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 8 & 10 & 6 & 1 & 7 & 2 & 9 & 4 & 3 & 5 \end{pmatrix}$$

