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Homework 13

10/24/2021

1. a)  $\arctan(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$   
 $\arctan(1) = \frac{\pi}{4} = \frac{1^1}{1} - \frac{1^3}{3} + \frac{1^5}{5} - \frac{1^7}{7} + \frac{1^9}{9} \dots = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \dots$   
 so  $\pi = 4(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \dots)$

b)  $\pi = 4(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \frac{1}{15} + \frac{1}{17} - \frac{1}{19} + \frac{1}{21})$   
 $\approx 3.232$

2a)  $\arctan x + \arctan y = \arctan\left(\frac{x+y}{1-xy}\right)$       $x = \frac{1}{2}$     $y = \frac{1}{3}$

$\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) = \arctan\left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{6}}\right) = \arctan\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right) = \arctan(1) = \frac{\pi}{4}$

b)  $\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$       $\pi = 4(\arctan \frac{1}{2} + \arctan \frac{1}{3})$

$\arctan \frac{1}{2} = \frac{(\frac{1}{2})^1}{1} - \frac{(\frac{1}{2})^3}{3} + \frac{(\frac{1}{2})^5}{5} - \frac{(\frac{1}{2})^7}{7} \dots = \frac{1}{2} - \frac{1}{24} + \frac{1}{160} - \frac{1}{896} \dots$

$\arctan \frac{1}{3} = \frac{(\frac{1}{3})^1}{1} - \frac{(\frac{1}{3})^3}{3} + \frac{(\frac{1}{3})^5}{5} - \frac{(\frac{1}{3})^7}{7} \dots = \frac{1}{3} - \frac{1}{81} + \frac{1}{1215} - \frac{1}{15309} \dots$

$\arctan \frac{1}{2} + \arctan \frac{1}{3} = \frac{6229}{13440} + 0.32 = 0.785$

$\pi = 4(0.785) = 3.14$

3.  $\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$       $\pi = 4(4\arctan \frac{1}{5} - \arctan \frac{1}{239})$

$\arctan \frac{1}{5} = \frac{(\frac{1}{5})^1}{1} - \frac{(\frac{1}{5})^3}{3} + \frac{(\frac{1}{5})^5}{5} \dots = \frac{1}{5} - \frac{1}{375} + \frac{1}{15625} \dots$

$\arctan \frac{1}{239} = \frac{(\frac{1}{239})^1}{1} - \frac{(\frac{1}{239})^3}{3} + \frac{(\frac{1}{239})^5}{5} \dots = \frac{1}{239} - \frac{1}{40955757} + 2.565 \times 10^{-13}$

$4\arctan(\frac{1}{5}) = 4\left(\frac{9253}{46875}\right) - \arctan\left(\frac{1}{239}\right) = 0.7854$

so  $\pi \approx 3.141$

4.  $13 - \left(\frac{1}{2}\right)^5 \frac{(20)(50)+13}{2^{10}} + \left(\frac{1.3}{2.4}\right)^5 \frac{(40)(91)+13}{2^{20}} - \left(\frac{1.3.5}{2.4.6}\right)^5 \frac{60.137+13}{2^{30}} \dots \approx \frac{129}{\pi^2}$

$\frac{129}{\pi^2} = 13 - \left(\frac{1}{32}\right)\left(\frac{1013}{1024}\right) + \left(\frac{243}{32768}\right)(3.484 \times 10^{-3}) - (2.48 \times 10^{-3})(7.359 \times 10^{-6})$

$13 - \frac{1013}{32768} + 2.584 \times 10^{-5} - (2.2 \times 10^{-8})$

$\frac{129}{\pi^2} \approx 12.969$

3.14161 only the first 4 digits