Quin Buob

HW 13 Ok to Post

1) $\tan^{-1}(x) = \sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+1} x^{2n+1}$ $\tan^{-1}(1) = \frac{\pi}{4}$

$\Rightarrow \frac{\pi}{4} = \tan^{-1}(1) = \sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+1} (1)^{2n+1} = \sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+1}$

$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \ldots$

$\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} - \ldots\right)$

1') $4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \frac{1}{15} + \frac{1}{17} - \frac{1}{19}\right)$

$= \frac{47028692}{14549535} = 3.23231$

error $= \frac{4}{21}$

2) $\tan^{-1}(A) + \tan^{-1}(B) = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$

$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{1/2 + 1/3}{1 - 1/3(1/2)}\right) = \tan^{-1}\left(\frac{5/6}{1 - 1/6}\right) = \tan^{-1}\left(\frac{5/6}{5/6}\right) = \tan^{-1}(1) = \frac{\pi}{4}$

2') $4\tan^{-1}(1) = 4\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = \pi$

$= 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \frac{1}{15} + \frac{1}{17} - \frac{1}{19} + \frac{1}{21} - \frac{1}{23} + \frac{1}{25} - \frac{1}{27}\right.$

$+ \frac{1}{29} - \frac{1}{31} + \frac{1}{33} - \frac{1}{35} + \frac{1}{37} - \frac{1}{39} + \frac{1}{41} - \frac{1}{43} + \frac{1}{45} - \frac{1}{47} + \frac{1}{49} - \frac{1}{51} + \frac{1}{53}$

$- \frac{1}{55} + \frac{1}{57} - \frac{1}{59} + \frac{1}{61} - \frac{1}{63} - \ldots$

3) $\pi/4 = 4\left(\frac{1}{5} - \frac{1}{3}\left(\frac{1}{5^3}\right) + \frac{1}{5}\left(\frac{1}{5^5}\right)\right) - \left(\frac{1}{239} - \frac{1}{3}\left(\frac{1}{239^3}\right) + \frac{1}{5}\left(\frac{1}{239^5}\right)\right)$

$\pi/4 = 0.9869866 - 0.004184$

$\pi/4 = 0.9808026$