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1. $\frac{\pi}{4} = \arctan(1)$

we know that $\arctan(x) = \int_0^x \frac{1}{1+t^2}\,dt$

when $x=1$

$\arctan(1) = \int_0^1 \frac{1}{1+t^2}\,dt = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\cdots$

so $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$

$\pi = 4(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots)$

2. $\arctan\frac{1}{2} + \arctan\frac{1}{3} = \arctan\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}} = \arctan(1) = \frac{\pi}{4}$

2'. $\arctan(x) = \int_0^x \frac{1}{1+t^2}\,dt$

when $|t|<1$

$\sum_{k=0}^{\infty} (-1)^k t^{2k}$ converges uniformly

$f(t) = \arctan(t) = \int_0^t \frac{1}{1+t^2}\,dt = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} t^{2n+1}$

so $\arctan(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$

3. $4\arctan\frac{1}{5} = 0.78950128$

$\arctan\frac{1}{239} = 0.004184$

$(4\arctan\frac{1}{5} - \arctan\frac{1}{239})\cdot 4 \approx 3.1417$

4. $13 - (\frac{1}{2})^5 \frac{20\cdot 50+13}{2^{10}} + (\frac{3}{8})^5 \frac{40\cdot 91+13}{2^{20}} - (\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6})^5 \frac{60\cdot 132+13}{2^{30}} = \frac{128}{\pi^2}$

$\pi = \sqrt{\frac{128}{13 - (\frac{1}{2})^5 \frac{20\cdot 50+13}{2^{10}} + \cdots}} = 3.17622$

3 digits agree