NAME: Tianyi He
E-MAIL ADDRESS: th586@scarletmail.rutgers.edu
It is OK to post the homework in your web-site

1. We know that $\arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots$

when $x=1$, $\arctan(1) = \frac{\pi}{4}$ since $\tan(\frac{\pi}{4}) = 1$

So $\pi = 4(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots)$

1'. $\pi = 4 \underbrace{- \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \frac{4}{13} - \frac{4}{15} + \frac{4}{17} - \frac{4}{19}}_{\text{The first 10 non-zero terms}} + \frac{4}{21}$

Error $\leq a_{10+1} \leq \frac{4}{21}$.

2. Let $A = \arctan(\frac{1}{2})$, $B = \arctan(\frac{1}{3})$, So $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - (\tan A)(\tan B)} = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}} = 1$$

Since $-\frac{\pi}{2} < A < \frac{\pi}{2}$, $0 < B < \frac{\pi}{4}$, then $A + B = \frac{\pi}{4}$,

We prove that $\arctan\frac{1}{2} + \arctan\frac{1}{3} = \frac{\pi}{4}$.

2'. $\arctan\frac{1}{2} + \arctan\frac{1}{3} = \frac{\pi}{4} = \arctan(1)$

$$\pi = 4\arctan(1) = 4\left[(-1)^0 + (-1)^1 \frac{1}{3} + (-1)^2 \frac{1}{5} + (-1)^3 \frac{1}{7} + \cdots\right]$$

$$= 4(1 \underbrace{- \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{101}}_{50 \text{ terms}} - \cdots)$$

$\frac{1}{101} \approx 0.0099$ (two digits after the decimal point)

So $\pi \approx \sum_{n=0}^{49} (-1)^n \frac{1^{2n+1}}{2n+1} \approx 3.12$

3. $\pi = 4(4\arctan\frac{1}{5} - \arctan\frac{1}{239}) \approx 3.14$

4.

$$13-\left(\frac{1}{2}\right)^5\frac{20\cdot 50+13}{2^{10}}+\left(\frac{1\cdot 3}{2\cdot 4}\right)^5\frac{40\cdot 91+13}{2^{20}}-\left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^5\frac{60\cdot 132+13}{2^{30}}\approx 12.96911$$

$$\pi\approx\sqrt{\frac{128}{12.96911}}\approx 3.14159$$

It at least agree with 3.14159.