HOMEWORK 13 : NINA CHALGERI

① $\tan\left(\frac{\pi}{4}\right) = 1 \qquad \arctan(1) = \frac{\pi}{4}$

$$\arctan(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}, \quad -1 \le x \le 1$$

$$\Rightarrow \arctan(1) = \sum_{n=0}^{\infty} (-1)^n \frac{(1)^{2n+1}}{2n+1} = \frac{\pi}{4}$$

REARRANGING:

$$\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots\right)$$

$$\frac{\pi}{4} = \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots\right)$$

~~which~~ since:

$$\arctan(1) = \frac{\pi}{4} = \sum_{n=0}^{\infty} (-1)^n \frac{(1)^{2n+1}}{2n+1} = \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots\right)$$

$\pi = \pi$ ✓

①'

$S = S_n + R_n$

error: $|S - S_n| = |R_n|$

$$S_{10} = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \frac{1}{15} + \frac{1}{17} - \frac{1}{19}\right)$$

≈ 3.04

$|\pi - S_{10}| = 0.099$

②

$$\frac{\pi}{4} = \arctan\frac{1}{2} + \arctan\frac{1}{3}$$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$\tan\left(\arctan\left(\tfrac{1}{2}\right) + \arctan\left(\tfrac{1}{3}\right)\right) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2}\cdot\frac{1}{3}} = \frac{\frac{5}{6}}{1 - \frac{1}{6}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

$\Rightarrow \tan\left(\arctan\left(\tfrac{1}{2}\right) + \arctan\left(\tfrac{1}{3}\right)\right)$

$= \tan(1)$?

$\Rightarrow \frac{\pi}{4}$

(2')

$$\frac{\pi}{4} = \sum_{n=0}^{\infty} (-1)^n \frac{\left(\frac{1}{2}\right)^{2n+1}}{2n+1} + \sum_{n=0}^{\infty} (-1)^n \frac{\left(\frac{1}{3}\right)^{2n+1}}{2n+1}$$

$$\pi \approx 4\left(\sum_{n=0}^{10} (-1)^n \frac{(1/2)^{2n+1}}{2n+1} + \sum_{n=0}^{\infty} (-1)^n \frac{(1/3)^{2n+1}}{2n+1} \right) = 3.14$$

$$\pi \approx 3.14$$

(3)

$$\frac{\pi}{4} = 4 \arctan \frac{1}{5} - \arctan \frac{1}{239}$$

$$\pi \approx 4\left(4 \cdot \sum_{n=0}^{10} (-1)^n \frac{(1/5)^{2n+1}}{2n+1} - \sum_{n=0}^{10} (-1)^n \frac{(1/239)^{2n+1}}{2n+1} \right)$$

$$\pi \approx 3.142$$

(4) approx. $\frac{128}{\pi^2}$

$$13 - \left(\frac{1}{5}\right)^5 \frac{20(50)+13}{2^{10}} + \left(\frac{1\cdot 3}{2\cdot 4}\right)^5 \frac{40-91+13}{2^{20}} - \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^5 \frac{60-132+13}{2^{30}}$$

$$= 12.99970925 \approx \frac{128}{\pi^2}$$

$$\Rightarrow \pi \approx 3.137\ldots$$

$\Rightarrow$ 2 digits