Homework 13

(1) Taylor series of arctan(x) $= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$

$x = 1: \frac{\pi}{4} = +1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \dots$

$\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \dots\right)$

(1') $S_{10} = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \frac{1}{15} + \frac{1}{17} - \frac{1}{19}\right)$

$= 3.0418$

$|S - S_{10}| = |\pi - S_{10}| = 0.09975\dots$

(2) $\frac{\pi}{4} = \arctan\frac{1}{2} + \arctan\frac{1}{3}$

$\tan(\arctan(1/2) + \arctan(1/3)) = \dfrac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2}\cdot\frac{1}{3}} = 1$

$\arctan(\tan(\arctan 1/2 + \arctan 1/3)) = \arctan(1) = \pi/4$

$\arctan\frac{1}{2} + \arctan\frac{1}{3} = \frac{\pi}{4}$

(2') $\frac{\pi}{4} = \sum_{n=0}^{\infty} \frac{(-1)^n (1/2)^{2n+1}}{2n+1} + \sum_{n=0}^{\infty} (-1)^n \frac{(1/3)^{2n+1}}{2n+1}$

$\pi \approx 4\left(4\sum_{n=0}^{10} (-1)^n \frac{(1/5)^{2n+1}}{2n+1} - \sum_{n=0}^{10} (-1)^n \frac{(1/239)^{2n+1}}{2n+1}\right)$

$\pi \approx 3.142$

(3) $n = 10$: Similar to above we get

$\pi \approx 4\left(4\cdot\sum_{n=0}^{10} (-1)^n \frac{(1/5)^{2n+1}}{2n+1} - \sum_{n=0}^{10} (-1)^n \frac{(1/239)^{2n+1}}{2n+1}\right)$

$\pi \approx 3.142$

(4') $13 - \left(\frac{1}{2}\right)^5 \frac{26(50)+13}{2^{16}} + \left(\frac{1\cdot 3}{2\cdot 4}\right)^5 \frac{40 - d_1 1 + 13}{2^{20}}$

$- \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^5 \frac{66 - 137 + 13}{2^{30}}$

$= 12.99970975 \approx \frac{128}{\pi^2}$

$\pi = 3.137893\ 252$

2 digits agree with π.