Larry Vo

$\frac{11}{6} + \frac{104}{61}$

H.W 13

1. $\tan(\frac{\pi}{4}) = 1$, $\arctan(1) = \frac{\pi}{4}$

$\arctan(x) = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \frac{1}{9}x^9 + \ldots$

$\arctan(1) = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \ldots = \frac{\pi}{4}$

$\pi = 4(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \ldots)$

First ten terms

$\pi \approx 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \frac{4}{13} - \frac{4}{15} + \frac{4}{17} - \frac{4}{19}$

≈ 3.04183

error is at most $\frac{4}{21}$

2. $\arctan(x) + \arctan(y) = \arctan\left(\frac{x+y}{1-xy}\right)$

$\arctan(\frac{1}{2}) + \arctan(\frac{1}{3}) = \arctan\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-(\frac{1}{2})(\frac{1}{3})}\right) = \arctan(1)$

$\arctan(\frac{1}{2}) + \arctan(\frac{1}{3}) = \frac{\pi}{4}$

$\pi = 4\arctan(\frac{1}{2}) + 4\arctan(\frac{1}{3})$

$= 4\sum_{n=0}^{\infty} (-1)^n \frac{(\frac{1}{2})^{2n+1}}{2n+1} + 4\sum_{n=0}^{\infty} (-1)^n \frac{(\frac{1}{3})^{2n+1}}{2n+1}$

$\approx 4(.46364) + 4(.32175)$

$\pi \approx 3.14$

3. $\frac{\pi}{4} = 4\arctan\frac{1}{5} - \arctan\frac{1}{239}$

$$\pi = 16\sum_{n=0}^{\infty} \frac{(-1)^n\left(\frac{1}{5}\right)^{2n+1}}{2n+1} - 4\sum_{n=0}^{\infty} \frac{(-1)^n\left(\frac{1}{239}\right)^{2n+1}}{2n+1}$$

$$\approx 16(.19739) - 4(.00418)$$

$$\pi \approx 3.141$$

4. $$13 - \left(\frac{1}{2}\right)^5 \frac{20\cdot 150+13}{2^{10}} + \left(\frac{1\cdot 3}{2\cdot 4}\right)^5 \frac{40\cdot 90+13}{2^{20}} - \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^5 \frac{60\cdot 132+13}{2^{30}}$$

$$\approx 12.96911 \approx \frac{128}{\pi^2}$$

$$\pi\sqrt{12.96911} \approx \sqrt{128}$$

$$\pi \approx \frac{\sqrt{128}}{\sqrt{12.96911}} \approx 3.14159283620474494912$$

$$\pi = 3.14159265358979 3 \ldots$$

Agrees up to the 6th decimal space.