

1)

$$\begin{aligned}
 \arctan(1) &= \frac{\pi}{4} \\
 \pi &= 4 * \arctan(1) \\
 &= 4 * [1 - x^2 + x^4 - x^6 + \dots] \\
 4 * \int [1 - x^2 + x^4 - x^6 + \dots] dx \\
 &= 4 * \left[x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \right] \\
 4 \left[1 - \frac{1^3}{3} + \frac{1^5}{5} - \frac{1^7}{7} + \dots \right] &\approx \pi
 \end{aligned}$$

1')

$$\begin{aligned}
 4 \left[1 - \left(\frac{1^3}{3}\right) + \left(\frac{1^5}{5}\right) - \left(\frac{1^7}{7}\right) + \left(\frac{1^9}{9}\right) - \left(\frac{1^{11}}{11}\right) + \left(\frac{1^{13}}{13}\right) - \left(\frac{1^{15}}{15}\right) + \left(\frac{1^{17}}{17}\right) - \left(\frac{1^{19}}{19}\right) \right] &= 3.04183961893 \\
 3.14159265359 - 3.04183961893 &= .09975303465
 \end{aligned}$$

2)

$$\begin{aligned}
 \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) &= \frac{\pi}{4} \\
 \tan(\alpha) &= \frac{1}{2}, \tan(\beta) = \frac{1}{3} \\
 \tan(\alpha + \beta) &= \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)} \\
 &= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} * \frac{1}{3}} \\
 &= \frac{\frac{5}{6}}{1 - \frac{1}{6}} = 1 \\
 \arctan(1) &= \frac{\pi}{4}
 \end{aligned}$$

2')

$$\begin{aligned}
 \pi &= 4 * \arctan(1) \\
 &= 4 * [1 - x^2 + x^4 - x^6 + \dots] \\
 4 * \int [1 - x^2 + x^4 - x^6 + \dots] dx \\
 &= 4 * \left[x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \right] \\
 4 \left[1 - \frac{1^3}{3} + \frac{1^5}{5} - \frac{1^7}{7} + \dots \right] \\
 &= 2.89
 \end{aligned}$$

3)

$$\frac{\pi}{4} = 4 \arctan\left(\frac{1}{5}\right) - \arctan\left(\frac{1}{239}\right)$$

$$= .785$$

4)

$$13 - \left(\frac{1}{5}\right)^5 * \frac{20 * 50 + 13}{2^{10}} + \left(\frac{1 * 1}{2 * 4}\right)^5 * \frac{40 * 91 + 13}{2^{20}} - \left(\frac{1 * 3 * 5}{2 * 4 * 6}\right)^5 * \frac{60 * 132 + 13}{2^{30}}$$

$$\frac{128}{\pi^2} = 12.99970$$

$$\pi^2 = \frac{128}{12.99970}$$

$$= 3.13789$$

$$\pi = 3.14159265$$

the approximation with my rounding shows the first 3 digits are close when rounding