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## HW 13

(1) $\frac{\pi}{4} = \arctan(1) = \int_0^1 \frac{1}{1+x^2}\,dx = \int_0^1 \left( \sum_{n=0}^{n} (-1)^n \cdot x^{2n} + \frac{(-1)^{n+1} \cdot x^{2n+2}}{1+x^2} \right) dx$

$= \sum_{n=0}^{n} \frac{(-1)^n}{2n+1} + (-1)^{n+1} \left( \int_0^1 \frac{x^{2n+2}}{1+x^2}\,dx \right) \to 0 \leq \int_0^1 \frac{x^{2n+2}}{1+x^2}\,dx \leq \int_0^1 x^{2n+2}\,dx = \frac{1}{2n+3} \to 0$

By squeeze theorem, $\frac{\pi}{4} = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}}$

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1.) $\frac{\pi}{4} = \arctan\frac{1}{2} + \arctan\frac{1}{3}$

$\arctan\frac{1}{2} = x,\ \tan x = \frac{1}{2}$
$\arctan\frac{1}{3} = y,\ \tan y = \frac{1}{3}$
$\Bigg\}\ \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$\Rightarrow \tan(A+B) = \frac{x+y}{1-xy} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$

$\tan(A+B) = 1 \rightarrow \tan(1) = \frac{\pi}{4}$

2.) Calculator answer: $\underline{2.67}$

3.) $\frac{\pi}{4} = 4\arctan\frac{1}{5} - \arctan\frac{1}{239}$

$\tan^{-1}x = x - \frac{x^3}{3!} + \frac{x^5}{5} + \cdots$

$\pi = 16\tan^{-1}\frac{1}{5} - 4\tan^{-1}\frac{1}{239} = \underline{3.141}$ Calculator Answer

4.) $\frac{128}{\pi^2} = 13 - \left(\frac{1}{2}\right)^5 \left(\frac{20\cdot 30+13}{2^{0}}\right) + \left(\frac{1\cdot 3}{2\cdot 4}\right)\left(\frac{40\cdot 91+13}{2^{21}}\right) - \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)\left(\frac{60\cdot 132+13}{2^{21}}\right) = \text{"}\underline{x}\text{"}$

$= \frac{128}{\pi^2} \cong x \rightarrow \sqrt{\frac{128}{x}} = \pi = 3.31608 \rightarrow$ Only the first term matches.