Homework 13.

1. $1/1-y = 1+y+y^2+\cdots$ $\arctan(x) = x - x^3/3 + x^5/5 - x^7/7 + \cdots$

substitute $y=-x^2$ for $x=1$ given $\arctan(1) = \pi/4$

$1/1+x^2 = 1-x^2+x^4-x^6+$ $\pi/4 = 1 - 1/3 + 1/5 - 1/7 \cdots$

$\int 1/1+x^2 = \int 1-x^2+x^4-x^6+\cdots$ $\pi = 4\cdot(1-1/3+1/5-1/7\cdots)$

2. $\arctan 1/2 + \arctan 1/3$

$= \arctan\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-1/6}\right)$

$= \arctan(1)$

$= \pi/4$

2' $\arctan(x+y) = \arctan\left(\frac{x+y}{1-xy}\right)$

Taylor $\arctan(x) = 0 + \frac{1}{1!}x + \frac{0}{2!}x^2 + \frac{-2}{3!}x^3 + \frac{0}{4!}x^4 + \frac{24}{5!}x^5 + \cdots$

$= x - 1/3x^3 + 1/5x^5 - 1/7x^7 + 1/9x^9$

$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$

3. $\pi/4 = 4\arctan 1/5 - \arctan 1/239$

$= (0.197)\cdot 4 - 0.004$

$= 0.784$

$\pi = 4\times 0.784 = 3.136$

4. $13 - (1/2)^5 \frac{20.50+13}{2^{10}} + \left(\frac{1\cdot 3}{2\cdot 4}\right)^5 \frac{40.91+13}{2^{20}} - \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^5 \frac{60.132+13}{2^{30}}$