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It is OK to post the homework in your web-site

1. $f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2$

$f(0) = \sin(0) = 0$, $f'(x) = \cos(x+x^2)(1+2x)$, $f'(0) = \cos(0) = 1$

$f''(x) = -\sin(x+x^2)(1+2x)^2 + 2\cos(x+x^2)$, $f''(0) = 2\cos(0) = 2$

$f(x) = 0 + x + x^2 = x + x^2$

2. $\sin(x+x^2) = (x+x^2) - \frac{1}{3!}(x+x^2)^3 + \frac{1}{5!}(x+x^2)^5 + \cdots$

$= x + x^2 - \frac{x^3+3x^4+\cdots}{6} + \cdots$

$= x + x^2 - \frac{x^3}{6} - \frac{x^4}{2} + \cdots$

3. $f(x) = \arctan(x)$, $f'(x) = \frac{1}{1+x^2}$.

Since $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, then $\frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty}(-x^2)^n = \sum_{n=0}^{\infty}(-1)^n x^{2n}$,

and $f'(x) = \sum_{n=0}^{\infty}(-1)^n x^{2n}$.

So $f(x) = \int \sum_{n=0}^{\infty}(-1)^n x^{2n}\,dx = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1} + C$.

Since $f(0) = \arctan(0) = 0$, $f(0) = \sum_{n=0}^{\infty}(-1)^n \cdot (0) + C = 0$, then $C = 0$.

So $f(x) = \sum_{n=0}^{\infty}(-1)^n \cdot \frac{x^{2n+1}}{2n+1}$.

4. Let $\alpha = \arctan x$ $\beta = \arctan y$, then $x = \tan\alpha$ $y = \tan\beta$.

So $\tan(\alpha+\beta) = \frac{xy}{1-xy}$, $\alpha+\beta = \arctan\left(\frac{xy}{1-xy}\right)$, $\arctan x + \arctan y = \arctan\left(\frac{xy}{1-xy}\right)$

5. Let $A = \arctan\left(\frac{1}{2}\right)$, $B = \arctan\left(\frac{1}{3}\right)$, So $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - (\tan A)(\tan B)} = \frac{\frac{1}{2}+\frac{1}{3}}{1 - \frac{1}{2}\times\frac{1}{3}} = 1$

Since $-\frac{\pi}{2} < A < \frac{\pi}{2}$, $0 < B < \frac{\pi}{4}$, then $A+B = \frac{\pi}{4}$,

We prove that $\arctan\frac{1}{2} + \arctan\frac{1}{3} = \frac{\pi}{4}$.