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HW 12 Ok to post

1) $x=0$ $f(x)=\sin(x+x^2)$ $f(0)=\sin(0)=0$

$f'(x)=(1+2x)\cos(x+x^2)$ $f'(0)=(1+2(0))\cos(0)=1$

$f''(x)=2\cos(x+x^2)-\sin(x+x^2)(1+2x)^2$ $f''(0)=2\cos(0)-\overset{0}{\cancel{\sin(0)}}(1)^2=2$

$f(x)=x+x^2$

2) $\sin z = z-\frac{1}{3!}z^3+\frac{1}{5!}z^5-\frac{1}{7!}z^7$

we want to know $\sin(x+x^2)$

$z=x+x^2$

$\sin(x+x^2)=x+x^2-\frac{1}{3!}(x+x^2)^3+\frac{1}{5!}(x-x^2)^5-\frac{1}{7!}(x+x^2)^7$

$=x+x^2-\frac{1}{6}x^3-\frac{1}{2}x^4-\frac{1}{2}x^5-\frac{1}{6}x^6+\frac{1}{120}x^5+\frac{1}{24}x^6+\frac{1}{12}x^7+\frac{1}{12}x^8+\frac{1}{24}x^9+\frac{1}{120}x^{10}$

$=x+x^2-\frac{1}{6}x^3-\frac{1}{2}x^4$

$\sin(x+x^2)=x+x^2-\frac{1}{6}x^3-\frac{1}{2}x^4$

3) We know $\tan^{-1}(x)=\int_0^x \frac{1}{1+t^2}dt$ and

$\sum_{n=0}^{\infty} z^n=\frac{1}{1-z}$ $|z|<1$

$z=-t^2$

$\frac{1}{1+t^2}=\sum_{n=0}^{\infty}(-t^2)^n=\sum_{n=0}^{\infty}(-1)^n t^{2n}$

$\tan^{-1}(x)=\int_0^x\frac{1}{1+t^2}dt=\int_0^x\sum_{n=0}^{\infty}(-1)^n t^{2n}=\int_0^x 1\,dt-\int_0^x t^2dt+\int_0^x t^4dt$

$=x-\frac{1}{3}x^3+\frac{1}{5}x^5\ldots$

$\tan^{-1}(x)=\sum_{n=0}^{\infty}(-1)^n\frac{1}{2n+1}x^{2n+1}$

4) $\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$

$a = \tan^{-1}(A) \quad b = \tan^{-1}(B) \Rightarrow A = \tan a \quad B = \tan b$

$\tan^{-1}(\tan(a+b)) = \tan^{-1}\left(\frac{\tan a + \tan b}{1 - \tan a \tan b}\right)$

$a+b = \tan^{-1}\left(\frac{\tan a + \tan b}{1 - \tan a \tan b}\right)$ Make substitutions

$\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$

5) $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{\overset{3/6}{\frac{1}{2}} + \overset{2/6}{\frac{1}{3}}}{1 - 1/2(1/3)}\right) = \tan^{-1}\left(\frac{5/6}{1 - 1/6}\right) = \tan^{-1}(1) = \pi/4$