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Okay

HW 12

1. $f(x) = \sin(x+x^2)$

$f(0) = \sin(0+0^2) = 0$

$f'(x) = \cos(x+x^2) \cdot (1+2x)$

$f'(0) = \cos(0) \cdot (1+2(0)) = 1$

$f''(x) = -\sin(x+x^2) \cdot (1+2x) \cdot (1+2x) + \cos(x+x^2) \cdot 2$

$f''(0) = 0+2 = 2$

$$f(x) \approx \frac{0}{0!}x^0 + \frac{1}{1!}x^1 + \frac{2}{2!}x^2$$

$f'''(x) = -12x\sin(x^2+x) - \cos(x^2+x)(2x+1)^3 - 6\sin(x^2+x)$

$f'''(0) = -1$

$$f(x) \approx x + x^2 - \frac{1}{6}x^3$$

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2. $\sin z = z - \frac{1}{3!} z^3 + \frac{1}{5!} z^5 - \frac{1}{7} z^7 + \ldots$

$\sin(x+x^2) = (x+x^2) - \frac{1}{3!}(x+x^2)^3 + \frac{1}{5!}(x+x^2)^5 - \frac{1}{7}(x+x^2)^7 \ldots$

3. $\arctan x = \int_0^x \frac{1}{1+t^2}\, dt$

$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2} = \frac{1}{1-(-x^2)}$

$\frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty} (-x^2)^n = \sum_{n=0}^{\infty} (-1)^n x^{2n}$

Now $\frac{d}{dx}(\arctan x) = \sum_{n=0}^{\infty} (-1)^n x^{2n}$

$\int \frac{d}{dx}(\arctan x) = \int \sum_{n=0}^{\infty} (-1)^n x^{2n}$

$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} + C$

Notice $\arctan(0) = 0$

$0 = \sum_{n=0}^{\infty} (-1)^n \frac{0^{2n+1}}{2n+1} + C$ thus $C = 0$

therefore $\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$

4. $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-(\tan\alpha)(\tan\beta)}$

let $\tan(\alpha) = x$ and $\tan(\beta) = y$
then $\alpha = \arctan(x)$ and $\beta = \arctan(y)$.

Now we have $\alpha+\beta = \arctan\left(\frac{\tan\alpha + \tan\beta}{1-(\tan\alpha)(\tan\beta)}\right)$

Using our subtitutions, we have

$\arctan(x) + \arctan(y) = \arctan\left(\frac{x+y}{1-xy}\right)$

5. Using the identity above,

$\arctan(\frac{1}{2}) + \arctan(\frac{1}{3}) = \arctan\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-(\frac{1}{2})(\frac{1}{3})}\right)$

$= \arctan\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right)$

$= \arctan(1)$

$= \frac{\pi}{4}$

Therefore

$\arctan(\frac{1}{2}) + \arctan(\frac{1}{3}) = \frac{\pi}{4}$

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