1)

$$x = 0$$

$$f(x) = sin(x + x^2)$$

$$\mathrm{f}'(\mathrm{x}) = \cos(\mathrm{x} + \mathrm{x}^2)\,(1 + 2\mathrm{x})$$

$$f''(x) = -sin(x + x\text{^}2)(1 + 2x)\text{^}2 + 2cos(x + x\text{^}2)$$

$$\sum_{n=0}^{2} \frac{f^n(0)}{n!} x^n$$

$$= \frac{(\sin(0 + 0^2))}{0!} * x^0 + \frac{\cos(0 + 0^2)\,(1 + 2(0))}{1!} * x^1 + \frac{(-\sin(0 + 0\text{^}2)(1 + 2(0))\text{^}2 + 2cos(0 + 0\text{^}2))}{2!} * x^2$$

$$= x^2 + x$$

2)

$$x = 0$$

$$f(0) = sin(x + x^2) = 0$$

$$\mathrm{f}'(0) = \cos(\mathrm{x} + \mathrm{x}^2)\,(1 + 2\mathrm{x}) = 1$$

$$f''(0) = -sin(x + x^2)(1 + 2x)^2 + 2cos(x + x^2) = 2$$

$$f(x) \approx \frac{0}{0!} x^0 + \frac{1}{1!} x^1 + \frac{2}{2!} x^2$$

$$f(x) \approx x + x^2$$

3)

$$f(x) = \arctan(x)$$

$$f'(x) = \frac{1}{1 + x^2}$$

$$\frac{1}{1 - z} = \sum_{n=0}^{\infty} z^n$$

$$\frac{1}{1 - z} \Rightarrow \frac{1}{1 - (-x^2)} = \sum_{n=0}^{N} b^n$$

$$= 1 - x^2 + x^4 - x^6 + \cdots$$

$$\int [1 - x^2 + x^4 - x^6 + \cdots]dx$$

$$= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$$

4)

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$\alpha + \beta = \arctan\left(\frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}\right.$$

$$\alpha = \arctan(x) \quad \Rightarrow \quad x = \tan(\alpha)$$

$$\beta = \arctan(y) \quad \Rightarrow \quad y = \tan(\beta)$$

$$\arctan(\mathrm{x}) + \arctan(\mathrm{y}) = \arctan\left(\frac{(\mathrm{x}+\mathrm{y})}{(1-\mathrm{xy})}\right)$$

5)

$$\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) = \frac{\pi}{4}$$

$$\tan(\alpha) = \frac{1}{2}, \tan(\beta) = \frac{1}{3}$$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} * \frac{1}{3}}$$

$$= \frac{\frac{5}{6}}{1 - \frac{1}{6}} = 1$$

$$\arctan(1) = \frac{\pi}{4}$$