Hw 12

(1.) $f(x) = \sum_{n=0}^{\infty} \frac{f^{n}(0)}{n!} x^n$,

$f(x) = \sin(x+x^2)$

$n = 0, 1, 2$

$$\begin{cases} f'(x) = \cos(x+x^2)(1+2x) \\ f''(x) = 2\cos(x^2+x) - (2x+1)^2 \cdot \sin(x^2+x) \rightarrow \\ f'''(x) = -6(2x+1) \cdot \sin(x^2+x) - (2x+1)^3 \cdot \cos(x^2+x) \end{cases}$$

$f(0) = \frac{\sin(0)}{0!} 1 = \frac{0}{1}(1) = \underline{0}$

$f'(0) = 2\cos(0) - (1)^2 \cdot \sin(0) = \underline{2}$

$n = \{0, 1, 2\} = \{0, 2, -1\}$

$f''(0) = -6(1) \cdot \sin(0) - (1)^3 \cdot \cos^2(0) = \underline{-1}$

(2.) $\sin z = z - \frac{1}{3!} z^3 + \frac{1}{5!} z^5 - \frac{1}{7!} z^7$

$f(x) = \sin(x+x^2)$

$= (x+x^2) - \frac{1}{3!}(x+x^2)^3 + \frac{1}{5!}(x+x^2)^5 - \frac{1}{7!}(x+x^2)^7 + \frac{1}{9!}(x+x^2)^9$

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(3.) $\arctan x = \int_0^x \frac{1}{1+t^2}\,dt$

$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \Rightarrow \frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty} (-x^2)^n = (-1)^n \underline{x^{2n}}$$

$f'(x) = \sum_{n=0}^{\infty} (-1)^n x^{2n}$, integrating : $\sum_{n=0}^{\infty} \int (-1)^n x^{2n}\,dx = \sum_{n=1}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} + C$

Since $f(0) = \arctan(0) = 0$, $C = 0$

$$f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

(4.) $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-(\tan\alpha)(\tan\beta)}$, using the identity: arctan both sides,

$= \alpha+\beta = \arctan\left(\frac{\tan\alpha + \tan\beta}{(1-\tan\alpha\cdot\tan\beta)}\right)$ Set $\alpha = \arctan x$, $\rightarrow x = \tan\alpha$

$\beta = \arctan y$, $y = \tan\beta$

Substitute into the formula, $\arctan x + \arctan y = \arctan\left(\frac{x+y}{1-xy}\right)$

(5.) $\arctan\frac{1}{2} + \arctan\frac{1}{3} = \frac{\pi}{4}$

Set $A = \arctan\frac{1}{2}$ $-\frac{\pi}{2} < A < \frac{\pi}{4}$, $\tan A = \frac{1}{2}$

$B = \arctan\frac{1}{3}$ $0 < B < \frac{\pi}{4}$, $\tan B = \frac{1}{3}$

$A+B = \frac{\pi}{4} \rightarrow \tan(A+B) = 1$, $\underline{\underline{A+B = \frac{\pi}{4}}}$