Homework 12

1. $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$

$f(x) = \sin(x + x^2)$

$= 0 + \frac{d/dx(\sin(x+x^2))(0)}{1!} x + \frac{d^2/dx^2(\sin(x+x^2))(0)}{2!} + \frac{d^3/dx^3(\sin(x+x^2))(0)}{3!} \cdots$

That is the first three terms.

2. $\sin z = z - \frac{1}{3!} z^3 + \frac{1}{5!} z^5 - \frac{1}{7!} z^7 + \ldots$

$\sin(x+x^2) = 0 - \frac{1}{3!} \sin^3(x+x^2) + \frac{1}{5!} \sin^5(x+x^2) - \frac{1}{7!} \sin^7(x+x^2)$

$d/dx(\sin(x+x^2))(0) : 1$, $d^2/dx^2(\sin(x+x^2))(0) : 2$, $d^3/dx^3(\sin(x+x^2))(0) : -1$

$d^4/dx^4(\sin(x+x^2))(0) : -12$, $d^5/dx^5(\sin(x+x^2))(0) : -59$

$= x + x^2 - \frac{1}{6} x^3 - \frac{1}{2} x^4 - \frac{59}{120} x^5 + \ldots$

3. $\arctan x = \int_0^x \frac{1}{1+t^2} dt$

taylor $\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n$

$= 1 + z + z^2 + z^3 + z^4 + \ldots$

$\frac{1}{\arctan x} = \int_0^x 1 + t^2 dt$

taylor $\frac{1}{1-z} = \arctan \frac{1}{z}$

4. $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - (\tan\alpha)(\tan\beta)}$

Let $a = \arctan x$ and $b = \arctan y$ then $x = \tan a$, $y = \tan b$

We have $\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$

$\tan(\arctan x + \arctan y) = \frac{x+y}{1-xy}$, $\arctan x + \arctan y = \arctan\left(\frac{x+y}{1-xy}\right)$

5. $\arctan \frac{1}{2} + \arctan \frac{1}{3}$

$= \arctan \frac{\frac{1}{2} + \frac{1}{3}}{1 - (\frac{1}{2} \cdot \frac{1}{3})} = \arctan \frac{\frac{5}{6}}{\frac{5}{6}} = \arctan 1 = \pi/4$