Sarah Catovic (ok to post) Homework 11 10/18/2021

1. $x^3-9x-28=0$ $x=u+v$

$(u+v)^3-9(u+v)-28=0$

$u^3+3u^2v+3uv^2+v^3-9(u+v)-28=0$

$u^3+v^3+3uv(u+v)-9(u+v)-28=0$

$u^3+v^3+(u+v)(3uv-9)-28=0$

Wishful thinking: $3uv-9=0$ so $uv=3 \Rightarrow u^3v^3=27$

Left with $u^3+v^3=28$

so $z^2-28z+27=0$ so $u^3=27$ and $v^3=1$

$(z-1)(z-27)=0$ $u=3$ and $v=1$

so $x=4$ is one root

$$\begin{array}{r} x^2+4x+7 \\ x-4\,\overline{)\,x^3-9x-28} \\ x^3-4x^2 \\ \hline 4x^2-9x-28 \\ 4x^2-16x \\ \hline 7x-28 \\ 7x-28 \\ \hline 0 \end{array}$$

need to find the roots of

$x^2+4x+7=0$

$\dfrac{-4\pm\sqrt{4^2-4(7)}}{2}=\dfrac{-4\pm\sqrt{-12}}{2}$

$=-2\pm i\sqrt{3}$

are the other two roots

2. $x^3-30x-133=0$ $x=u+v$

$(u+v)^3-30(u+v)-133=0$

$u^3+v^3+3uv(u+v)-30(u+v)-133=0$

$u^3+v^3+(u+v)(3uv-30)-133=0$

Wishful thinking: $3uv-30=0$ so $uv=10 \Rightarrow u^3v^3=1000$

Left with $u^3+v^3=133$

so $z^2-133z+1000=0$ $(z-125)(z-8)=0$

so $z=125$ and $z=8$

$u^3=125$ and $v^3=8$ so $u=5$ and $v=2$

$x=7$ is one root

$$\begin{array}{r} x^2+7x+19 \\ x-7\,\overline{)\,x^3-30x-133} \\ x^3-7x^2 \\ \hline 7x^2-30x-133 \\ 7x^2-49x \\ \hline 19x-133 \\ \hline 0 \end{array}$$

other two solutions are roots of $x^2+7x+19=0$

$\dfrac{-7\pm\sqrt{49-4(19)}}{2}$

$\dfrac{-7\pm i3\sqrt{3}}{2}$ are the two other roots

3. $x^3 + px + q = 0$ $\quad x = u+v$

$(u+v)^3 + p(u+v) + q = 0$

$u^3 + v^3 + 3uv(u+v) + p(u+v) + q = 0$

$u^3 + v^3 + (u+v)(3uv + p) + q = 0$

Wishful thinking: $3uv + p = 0$ so $uv = \frac{-p}{3} \Rightarrow u^3v^3 = \frac{-p^3}{27}$

leaving $u^3 + v^3 = -q$

so $z^2 + qz - \frac{p^3}{27} = 0$ $\quad z = \frac{-q \pm \sqrt{q^2 + \frac{4p^3}{27}}}{2}$ $\quad u^3 = \frac{-q + \sqrt{q^2 + \frac{4p^3}{27}}}{2}$

$v^3 = \frac{-q - \sqrt{q^2 + \frac{4p^3}{27}}}{2}$ so $x = \left(\frac{-q + \sqrt{q^2 + \frac{4p^2}{27}}}{2}\right)^{1/3} + \left(\frac{-q - \sqrt{q^2 + \frac{4p^2}{27}}}{2}\right)^{1/3}$

To get the other two solutions, divide $x^3 + pq\, x q$ by the root being as above, and you'll get a quadratic. Use the quadratic formula to solve the quadratic and get the other two roots.

4. $x^3 + 3x^2 + 5x - 100 = 0$ $\quad y = x + 1$ so $x = y - 1$

$(y-1)^3 + 3(y-1)^2 + 5(y-1) - 100 = 0$

$(y-1)(y^2 - 2y + 1) + 3(y^2 - 2y + 1) + 5y - 5 - 100 = 0$

$y^3 - \cancel{3y^2} + 3y - 1 + \cancel{3y^2} - 6y + 3 + 5y - 5 - 100 = 0$

$\underline{y^3 - 3y - 103 = 0}$

this would be the reduced equation needed to be solved.