Quin Buob
HW 11 Ok to Post

D $x^3 - 9x - 28 = 0$

$x = u + v$

$(u+v)^3 - 9(u+v) - 28$

$u^3 + v^3 + 3uv^2 + 3u^2v - 9(u+v) - 28$

$u^3 + v^3 + 3uv(u+v) - 9(u+v) - 28$

$u^3 + v^3 + (3uv - 9)(u+v) - 28$

let $3uv - 9 = 0$

$uv = 3$

$\rightarrow u^3 + v^3 - 28 = 0$

$u^3 + v^3 = 28$

$u^3 v^3 = 27$

$X^2 - 28X + 27 = 0$

$x = \frac{28 \pm \sqrt{28^2 - 4(27)}}{2} = [27, 1] = [u^3, v^3]$

$[u, v] = [\sqrt[3]{27}, \sqrt[3]{1}] = [3, 1]$

$x_1 = 3 + 1 = 4$

$x_2 = \omega u + \omega^2 v = \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)(3) + \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)(1)$

$= -\frac{3}{2} + \frac{3\sqrt{3}}{2}i - \frac{1}{2} - \frac{\sqrt{3}}{2}i = -2 + i\sqrt{3}$

$x_3 = \omega^2 u + \omega v = \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)(3) + \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)(1) = -2 - i\sqrt{3}$

$[x_1, x_2, x_3] = [4, -2 + i\sqrt{3}, -2 - i\sqrt{3}]$

2) $x^3 - 30x - 133 = 0$

$x = u + v$

$u^3 + v^3 + 3uv(u+v) - 30(u+v) - 133 = 0$

$u^3 + v^3 + (3uv - 30)(u+v) - 133 = 0$

$3uv - 30 = 0$, $uv = 10$, $u^3 v^3 = 1000$

$u^3 + v^3 = 133$

$X^2 - 133x + 1000 = 0 \Rightarrow [125, 8] = [u^3, v^3]$

$[u, v] = [5, 2]$

$x_1 = 5 + 2 = 7$

$x_2 = \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)(5) + \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)(2) = -\frac{7}{2} + i\frac{3\sqrt{3}}{2}$

$x_3 = \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)(5) + \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)(2) = -\frac{7}{2} - i\frac{3\sqrt{3}}{2}$

3) $x^3 + px + q = 0$

$x = u + v$

$u^3 + v^3 + 3u^2v + 3uv^2 + p(u+v) + q = 0$

$u^3 + v^3 + 3uv(u+v) + p(u+v) + q = 0$

$u^3 + v^3 + (3uv + p)(u+v) + q = 0$

$3uv + p = 0 \Rightarrow uv = -\frac{p}{3} \Rightarrow u^3 v^3 = -\frac{p^3}{27}$

$u^3 + v^3 + q = 0 \Rightarrow u^3 + v^3 = -q$

$X^2 + \frac{p^3}{27} - q = 0$ $\qquad X = [u^3, v^3]$

$X = \frac{-\frac{p^3}{27} \pm \sqrt{\left(\frac{p^3}{27}\right)^2 + 4q}}{2} = \frac{-\frac{p^3}{27} \sqrt{\frac{p^6}{729} - 4q}}{2} = [u^3 + v^3]$

$x' = u + v$

$x^2 = \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)u + \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)v$

4) $x^3+3x^2+5x-100=0$

$y=x+\frac{3}{3}=x+1 \Rightarrow x=y-1$

$(y-1)^3+3(y-1)^2+5(y-1)-100$

$y^3+2y-103$

$y=u+v$

$u^3+v^3+(3uv+2)(u+v)-103=0$

$3uv+2=0 \Rightarrow uv=-\frac{2}{3} \Rightarrow u^3v^3=-\frac{8}{27}$

$u^3+v^3=103$

$X^2-103X-\frac{8}{27}=0$

$X=\frac{103 \pm \sqrt{103^2-\frac{32}{27}}}{2}$ $\quad [u^3, v^3]=\left[\frac{103+\sqrt{10618}}{2}, \frac{103-\sqrt{10618}}{2}\right]$

$y_1=u+v=\sqrt[3]{\frac{103+\sqrt{10618}}{2}}+\sqrt[3]{\frac{103-\sqrt{10618}}{2}}$

$y_2=\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)u+\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)v$

$y_3=\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)u+\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)v$

$x_1=y_1-1$

$x_2=y_2-1$

$x_3=y_3-1$