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Okay

HW 11

1. $x^3 - 9x - 28 = 0$

$x = u+v$

$(u+v)^3 - 9(u+v) - 28 = 0$

$u^3 + 3u^2v + 3uv^2 + v^3 - 9(u+v) - 28 = 0$

$u^3 + v^3 + 3uv(u+v) - 9(u+v) - 28 = 0$

$u^3 + v^3 + (3uv - 9)(u+v) - 28 = 0$

Want $3uv - 9 = 0$ so $uv = 3 \rightarrow u^3v^3 = 27$

$u^3 + v^3 = 28$

By lemma we have $x^2 - (u^3+v^3)x + u^3v^3 = 0$

or $x^2 - 28x + 27 = 0$

$u^3 = 27 \quad v^3 = 1$

$u = 3 \quad v = 1$

$x = 3+1 = 4$ is one root

Second root: $\omega u + \omega^2 v = \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)3 + \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)1$

$= -\frac{3}{2} + i\frac{3\sqrt{3}}{2} - \frac{1}{2} - i\frac{\sqrt{3}}{2} = -2 + \sqrt{3}i$

third root: $\omega^2 u + \omega v = 3\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) + 1\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$= -\frac{3}{2} - i\frac{3\sqrt{3}}{2} - \frac{1}{2} + i\frac{\sqrt{3}}{2} = -2 - \sqrt{3}i$

2. $x^3 - 30x - 133 = 0$

let $x = (u+v)$

$u^3 + v^3 + (3uv - 30)(u+v) - 133 = 0$

Want $3uv - 30 = 0$ so $uv = 10 \to u^3v^3 = 1000$

$u^3 + v^3 = 133$

$x^2 - 133x + 1000 = 0$

$u^3 = 125 \quad v^3 = 8$

$u = 5 \quad v = 2$

$x = 5 + 2 = 7$ is one root

Second root: $5\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) + 2\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$

$= -\frac{5}{2} + i\frac{5\sqrt{3}}{2} - 1 - i\frac{2\sqrt{3}}{2}$

$= -\frac{7}{2} + \frac{3\sqrt{3}}{2}i$

third root: $5\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) + 2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$= -\frac{5}{2} - i\frac{5\sqrt{3}}{2} - \frac{2}{2} + i\frac{2\sqrt{3}}{2}$

$= -\frac{7}{2} - \frac{3\sqrt{3}}{2}i$

3. $x^3+px+q=0$

let $x=(u+v)$

$u^3+3u^2v+3uv^2+v^3+p(u+v)+q=0$

$u^3+v^3+(3uv+p)(u+v)+q=0$

Want $3uv+p=0$ So $uv=\frac{-p}{3} \rightarrow u^3v^3=\frac{-p^3}{27}$

$u^3+v^3=-q$

$$x^2-(-q)x+\left(-\frac{p^3}{27}\right)=x^2+qx-\frac{p^3}{27}=0$$

$$u^3,v^3=\frac{-(-q)\pm\sqrt{(-q)^2-4(1)\left(-\frac{p^3}{27}\right)}}{2(1)}$$

$$u^3=\frac{-q+\sqrt{q^2+\frac{4p^3}{27}}}{2},\quad v^3=\frac{-q-\sqrt{q^2+\frac{4p^3}{27}}}{2}$$

$$u=\sqrt[3]{\frac{-q+\sqrt{q^2+\frac{4p^3}{27}}}{2}}\quad v=\sqrt[3]{\frac{-q-\sqrt{q^2+\frac{4p^3}{27}}}{2}}$$

$$x=u+v=\sqrt[3]{\frac{-q+\sqrt{q^2+\frac{4p^3}{27}}}{2}}+\sqrt[3]{\frac{-q-\sqrt{q^2+\frac{4p^3}{27}}}{2}}$$

Second root: $\omega u+\omega^2 v$

$$=\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\sqrt[3]{\frac{-q+\sqrt{q^2+\frac{4p^3}{27}}}{2}}+\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)\sqrt[3]{\frac{-q-\sqrt{q^2+\frac{4p^3}{27}}}{2}}$$

third root: $\omega^2 u+\omega v$

$$\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)\sqrt[3]{\frac{-q+\sqrt{q^2+\frac{4p^3}{27}}}{2}}+\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\sqrt[3]{\frac{-q-\sqrt{q^2+\frac{4p^3}{27}}}{2}}$$

4. $x^3 + 3x^2 + 5x - 100 = 0$

let $x = y - \frac{3}{3} = y - 1$

$(y-1)^3 + 3(y-1)^2 + 5(y-1) - 100 = 0$

$y^3 - 3y^2 + 3y - 1 + 3y^2 - 6y + 3 + 5y - 5 - 100 = 0$

$= y^3 + 2y - 103 = 0$

Solve for y and then to get x, you just subtract 1.