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1. $x^3 - 9x - 28 = 0$
   $x = u + v$
   $(u+v)^3 - 9(u+v) - 28 = 0$
   $u^3 + 3u^2v + 3uv^2 + v^3 - 9(u+v) - 28 = 0$
   $u^3 + v^3 + 3uv(u+v) - 9(u+v) - 28 = 0$
   $u^3 + v^3 + 3(u+v)(uv - 3) - 28 = 0$
   If $uv - 3 = 0$, $uv = 3$ and $u^3v^3 = 27$.
   That would also yield $u^3 + v^3 = 28$.
   $X^2 - 28X + 27 = 0$.
   $X = \frac{28 \pm \sqrt{28^2 - 4*27}}{2} = 1, 27$
   So $u = 1, v = 3$ (or vice versa).

   Root 1: $3 + 1 = 4$
   Root 2: $1(-\frac{1}{2} + \frac{i\sqrt{3}}{2}) + 3(-\frac{1}{2} - \frac{i\sqrt{3}}{2}) = -2 - i\sqrt{3}$
   Root 3: $3(-\frac{1}{2} + \frac{i\sqrt{3}}{2}) + 1(-\frac{1}{2} - \frac{i\sqrt{3}}{2}) = -2 + i\sqrt{3}$

2. $x^3 - 30x - 133 = 0$
   $x = u + v$
   $(u+v)^3 - 30(u+v) - 133 = 0$
   $u^3 + 3u^2v + 3uv^2 + v^3 - 30(u+v) - 133 = 0$
   $u^3 + v^3 + 3uv(u+v) - 30(u+v) - 133 = 0$
   $u^3 + v^3 + 3(u+v)(uv - 10) - 133 = 0$
   If $uv - 10$ were to equal 0, $uv = 10$, or $u^3v^3 = 1000$.
   This would also yield $u^3 + v^3 = 133$.
   $X^2 - 133X + 1000 = 0$
   $X = \frac{133 \pm \sqrt{133^2 - 4000}}{2} = \frac{133 \pm 117}{2} = 125, 8$
   So $u = 5, v = 2$ (or vice versa).

   Root 1: $5 + 2 = 7$
   Root 2: $5(-\frac{1}{2} + \frac{i\sqrt{3}}{2}) + 2(-\frac{1}{2} - \frac{i\sqrt{3}}{2}) = -\frac{7}{2} + \frac{3i\sqrt{3}}{2}$
   Root 3: $2(-\frac{1}{2} + \frac{i\sqrt{3}}{2}) + 5(-\frac{1}{2} - \frac{i\sqrt{3}}{2}) = -\frac{7}{2} - \frac{3i\sqrt{3}}{2}$

3. $x^3 + px + q = 0$.
   $x = u + v$
   $(u+v)^3 + p(u+v) + q = 0$
   $u^3 + v^3 + (u+v)(3uv + p) + q$
   $uv = -p/3, u^3v^3 = -p^3/27$
   $u^3 + v^3 = -q$
   $27X^2 + 27qX - p^3 = 0$
   $X = \frac{-27q \pm \sqrt{3^6q^2 + 4(3^3)p^3}}{54} = \frac{-9q \pm \sqrt{81q^2 + 12p^3}}{18}$
   $u = \sqrt[3]{\frac{-9q + \sqrt{81q^2 + 12p^3}}{18}}, v = \sqrt[3]{\frac{-9q - \sqrt{81q^2 + 12p^3}}{18}}$

   Root 1: $\sqrt[3]{\frac{-9q + \sqrt{81q^2 + 12p^3}}{18}} + \sqrt[3]{\frac{-9q - \sqrt{81q^2 + 12p^3}}{18}}$
   Root 2: $\sqrt[3]{\frac{-9q + \sqrt{81q^2 + 12p^3}}{18}}(-\frac{1}{2} + \frac{i\sqrt{3}}{2}) + \sqrt[3]{\frac{-9q - \sqrt{81q^2 + 12p^3}}{18}}(-\frac{1}{2} - \frac{i\sqrt{3}}{2})$
   Root 3: $\sqrt[3]{\frac{-9q - \sqrt{81q^2 + 12p^3}}{18}}(-\frac{1}{2} + \frac{i\sqrt{3}}{2}) + \sqrt[3]{\frac{-9q + \sqrt{81q^2 + 12p^3}}{18}}(-\frac{1}{2} - \frac{i\sqrt{3}}{2})$

4. $x^3 + 3x^2 + 5x - 100 = 0$
   $x = y - 1$
   $(y-1)^3 + 3(y-1)^2 + 5(y-1) - 100 = 0$
   $y^3 - 3y^2 + 3y - 1 + 3y^2 - 6y + 3 + 5y - 5 - 100 = 0$
   $y^3 + 2y - 103 = 0$