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1.  $p = -9$   $q = -28$

Let  $x = u + v$

$$u^3 + v^3 + (u+v)(3uv - 9) - 28 = 0$$

$$u^3 + v^3 = 28$$

$$u^3 v^3 = \frac{9^3}{27} = 27$$

$$u^3 = 14 + \sqrt{\frac{28^2}{4} + \frac{(-9)^3}{27}} = 14 + \sqrt{196 - 27} = 27$$

$$u = 3$$

$$v^3 = 14 - 13 = 1 \quad v = 1$$

So  $u + v = 4$

$$wu + w^2 v = -2 + \sqrt{3}i$$

$$w^2 u + wv = -2 - \sqrt{3}i$$

2.  $p = -30$   $q = -133$

$$u^3 = 15 + \sqrt{\frac{133^2}{4} - \frac{30^3}{27}} \quad v^3 = 15 - \sqrt{\frac{133^2}{4} - \frac{30^3}{27}}$$

$$u^3 + v^3 = 133 \quad \text{so } u + v = 7$$

$$u^3 v^3 = 1000 \quad u = 5 \quad v = 2$$

$$u + v = 7 \quad wu + w^2 v = -\frac{7}{2} + \frac{3\sqrt{5}}{2}i \quad w^2 u + wv = -\frac{7}{2} - \frac{3\sqrt{5}}{2}i$$

3.  $u^3 = -\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}} \quad v^3 = -\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}$

$$u^3 + v^3 = -9$$

$$u^3 v^3 = -\frac{p^3}{27}$$

$$u + v = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

4.  $y = x + \frac{3}{3} = x + 1$

$$(y-1)^3 + 3(y-1)^2 + 5(y-1) - 100 = 0$$

$$y^3 + 2y - 103 = 0$$