Vivian Choong
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Homework 11

(1) Use Cardano's method to find all three roots of the cubic equation

$x^3 - 9x - 28 = 0$

$x = u + v$

$(u+v)^3 - 9(u+v) - 28 = 0$

$u^3 + 3u^2v + 3uv^2 + v^3 - 9(u+v) - 28 = 0$

$u^3 + v^3 + (3u^2v + 3uv^2 - 9(u+v)) - 28 = 0$

$u^3 + v^3 + (3uv(u+v) - 9(u+v)) - 28 = 0$

$u^3 + v^3 + (u+v)(3uv - 9) - 28 = 0$

$3uv - 9 = 0$

$3uv = 9$

$uv = 3$

$u^3 + v^3 = 28$

$u^3v^3 = 27$

$x^2 - 28x + 27 = 0$

$(x-27)(x-1) = 0$

$x = 27, 1$

$u^3 = 27 \Rightarrow u = 3$

$v^3 = 1 \Rightarrow v = 1$

$x = u + v = 27 + 1 = \boxed{28}$

$\omega u + \omega^2 v = \left(\frac{-1}{2} + \frac{i\sqrt{3}}{2}\right)(3) + \left(\frac{-1}{2} - \frac{i\sqrt{3}}{2}\right)(1) \rightarrow -\frac{4 + 2i\sqrt{3}}{2} \rightarrow \boxed{-2 + i\sqrt{3}}$

$\omega^2 u + \omega v = \left(\frac{-1}{2} - \frac{i\sqrt{3}}{2}\right)(3) + \left(\frac{-1}{2} + \frac{i\sqrt{3}}{2}\right)(1) \rightarrow \frac{-4 - 2i\sqrt{3}}{2} \rightarrow \boxed{-2 - i\sqrt{3}}$

(2) Use Cardano's method to find all 3 roots of the cubic equation:

$x^3 - 30x - 133 = 0$

$x = u + v$

$(u+v)^3 - 30(u+v) - 133 = 0$

$u^3 + 3u^2v + 3uv^2 + v^3 - 30(u+v) - 133 = 0$

$u^3 + v^3 + 3uv(u+v) - 30(u+v) - 133 = 0$

$u^3 + v^3 + (3uv - 30)(u+v) - 133 = 0$

$3uv - 30 = 0$

$3uv = 30$

$uv = 10$

$u^3 + v^3 = 133$

$u^3v^3 = 1000$

$x^2 - 133x + 1000 = 0$

$(x - 125)(x - 8) = 0$

$x = 125, 8$

$/ = u + v = 125 + 8 = \boxed{133}$

$\left(\frac{-1}{2} + \frac{i\sqrt{3}}{2}\right)125 + \left(\frac{-1}{2} - \frac{i\sqrt{3}}{2}\right)8 = \boxed{\frac{-133 + 117i\sqrt{3}}{2}}$

$\left(\frac{-1}{2} - \frac{i\sqrt{3}}{2}\right)125 + \left(\frac{-1}{2} + \frac{i\sqrt{3}}{2}\right)8 = \boxed{\frac{-133 - 117i\sqrt{3}}{2}}$

(3) Derive a formula for the general cubic equation:

$$x^3 + px + q = 0$$

$$x = -q$$

$$x = \frac{q \pm (q-p)i\sqrt{3}}{2}$$