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Can Show

1)

$$x^3 - 9x - 28 = 0$$
$$u^3 + v^3 + (u+v)(3uv-9) - 28 = 0$$
$$3uv - 9 = 0$$
$$uv = 3$$
$$u^3v^3 = 27$$
$$u^3 + v^3 = 28$$
$$X^2 - 28X + 27$$
$$X = 1,27$$
$$u^3 = 1, v^3 = 27$$
$$u = 1, v = 3$$
$$x = u + v$$
$$x = 4$$

$$1\left(-\frac{1}{2} + i * \frac{\sqrt{3}}{2}\right) + 3\left(-\frac{1}{2} - i * \frac{1\sqrt{3}}{2}\right), 3\left(-\frac{1}{2} + i * \frac{\sqrt{3}}{2}\right) + 1\left(-\frac{1}{2} - i * \frac{\sqrt{3}}{2}\right)$$

$$x = -2 - \sqrt{3}i, -2 + \sqrt{3}i$$

2)

$$x^3 - 30x - 133$$

$$u^3 + v^3 + (u+v)(3uv-30) - 133 = 0$$

$$3uv - 30 = 0$$
$$uv = 10$$
$$u^3v^3 = 1000$$
$$u^3 + v^3 = 133$$
$$X^2 - 133X - 1000$$
$$X = 8,125$$
$$u^3 = 8, v^3 = 125$$
$$u = 2, v = 5$$
$$x = 7$$

$$x = 2\left(-\frac{1}{2} + i * \frac{\sqrt{3}}{2}\right) + 5\left(-\frac{1}{2} - i * \frac{\sqrt{3}}{2}\right), 2\left(-\frac{1}{2} - i * \frac{\sqrt{3}}{2}\right) + 5\left(-\frac{1}{2} + i * \frac{\sqrt{3}}{2}\right)$$

$$x = -3.5 + \frac{3\sqrt{3}i}{2}, -3.5 - \frac{3\sqrt{3}i}{2}$$

3)

$$x^3 + px + q$$
$$u^3 + v^3 + (u+v)(3uv+p) + q = 0$$
$$uv = -\frac{p}{3}$$

Can Show

$$u^3v^3 = -\frac{p^3}{3^3}$$

$$u^3 + v^3 = -q$$

$$X^2 - qX + p = 0$$

$$X = a, b$$

$$u = a, v = b$$

$$x = a^3 + b^3$$

$$a\left(-\frac{1}{2} + i * \frac{\sqrt{3}}{2}\right) + b\left(-\frac{1}{2} - i * \frac{\sqrt{3}}{2}\right), a\left(-\frac{1}{2} - i * \frac{\sqrt{3}}{2}\right) + b\left(-\frac{1}{2} + i * \frac{\sqrt{3}}{2}\right)$$

4)

$$x^3 + 3x^2 + 5x - 100 = 0$$

$$a = 1, b = 3, c = 5, d = -100$$

$$x = y - \frac{b}{3a}$$

$$x = y - 1$$

$$(y - 1)^3 + 3(y - 1)^2 + 5(y - 1) - 100 = 0$$

$$y^3 - 3y^2 + 3y - 1 + 3(y^2 - 2y + 1) + 5(y - 1) - 100$$

$$y^3 - 3y^2 + 3y^2 + 3y - 6y + 5y - 1 + 3 - 5 - 100$$

$$y^3 + 2y - 103$$

convert y to x

$$x^3 + 2x - 103$$