10/17/24

HW 11

1. Cardano's Method: $x^3 - 9x - 28 = 0$, $x^3 + px = q$

$x = u - v$, $3uv = p$, $u^3 - v^3 = q$

$x^3 - 9x = 28$: $\begin{cases} u^3 - v^3 = 28 \\ 3uv = -9 \end{cases} \rightarrow uv = -3$, $u^3 v^3 = -27$

So, $(u^3 - v^3)^2 = 28^2 \rightarrow u^6 - 2u^3v^3 + v^6 = 784$

↳ $x = u - v$, $3uv = p$, $u^3 - v^3 = q \rightarrow x = u - v$, $3uv = -9$, $u^3 - v^3 = 28$

$uv = -3$, $u^3 v^3 = -27 \rightarrow 4u^3v^3 = -108$

$(u^3 - v^3)^2 = 28^2 \rightarrow u^6 - 2u^3v^3 + 4u^3v^3 + v^6 = (784) + (-108)$

↳ $u^6 + 2u^3v^3 + v^6 = 676 \Rightarrow (u^3 + v^3)^2 = \sqrt{676} = 26$

$(u^3 + v^3) = 26$

$(u^3 - v^3) = 28$ $\Rightarrow$ $2u^3 = 54 \rightarrow u^3 = 27 \rightarrow \boxed{u = 3}$, $\boxed{v = -1}$

$x = u - v = 3 - (-1) = \boxed{4}$

$u + v = 3 + (-1) = \boxed{2}$

$\omega u + \omega^2 v = \left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)(3) + \left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)(-1) = \left(-\frac{3}{2} + \frac{3i\sqrt{3}}{2}\right) + \left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) = \boxed{-1 + \frac{4i\sqrt{3}}{2}}$

$\omega^2 u + \omega v = \left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)(3) + \left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)(-1) = \left(-\frac{3}{2} - \frac{3i\sqrt{3}}{2}\right) + \left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) = \boxed{-1 - \frac{4i\sqrt{3}}{2}}$

10/11/24 HW11

(2.) $x^3 - 30x = 133$

$x = v - v_1$

$v^3 - v^3 = 133$

$3uv = 30 \rightarrow uv = -10$

$u^3v^3 = -1000$

$(u^3 - v^3)^2 = (133)^2 = 17,689$, $4u^3v^3 = -4000$

$u^6 - 2u^3v^3 + 4u^3v^3 + v^6 = 17,689$

$u^6 + 2u^3v^3 + v^6 = 13,689 \quad = (u^3 + v^3)^2 = 13,689 \quad = u^3 + v^3 = \sqrt{13,689} = 117$

$u^3 + v^3 = 117$

$u^3 - v^3 = 133$

$u^3 = 250 \rightarrow u = \sqrt[3]{250} = \sqrt[3]{125 \cdot 2} = \boxed{5\sqrt[3]{2}} = u$

$2v^3 = -16 \rightarrow v^3 = -8 \rightarrow \boxed{v = -2}$

$u + v = \boxed{(-2) + (5\sqrt[3]{2})}$

$\omega u + \omega^2 v = \left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)(5\sqrt[3]{2}) + \left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)(-2) = \boxed{\left(\frac{5\sqrt[3]{2}}{-2}\right) + \left(\frac{(5\sqrt[3]{2})(i\sqrt{3})}{2}\right) + (1 + i\sqrt{3})}$

$\omega^2 u + \omega v = \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)(5\sqrt[3]{2}) + \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)(-2)$

$y^2 - 2y + 1 (y-1)$

(3) $x^3 + px + q = 0$

$x = v - v$, $4u^3v^3 = 4\frac{p^3}{27}$

$u^3 - v^3 = q$ | $(u^3 - v^3)^2 = q^2 \rightarrow (u^6 - 2u^3v^3 + v^6) + (4u^3v^3) = 4\frac{p^3}{27} + q^2$

$3uv = p$ | $(u^3 + v^3)^2 = \left(4\frac{p^3}{27} + q^2\right)$

$uv = \frac{p}{3}$ $(u^3 + v^3) = \sqrt{4\frac{p^3}{27} + q^2}$

$(y^2 - 2y + 1)(y - 1)$

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(4.) $x^3 + 3x^2 + 5x - 100 = 0$

$y = x + \frac{a}{3}$, $x = y - \frac{b}{3a} \rightarrow x = y - 1$

$\rightarrow (y-1)^3 + 3(y-1)^2 + 5(y-1) = 100 \rightarrow y^3 - y^2 - 2y^2 + 2y + y - 1) + (3y^2 - 6y + 3) + 5y - 5 = 100$

$y^3 + 2y - 3 = 100 \Rightarrow \boxed{y^3 + 2y = 97}$

$y = x + \frac{a}{3}$

$x(x^2 + 3x + 5)$

$x^3 + 5x = 3x^2$

$x(u^3 + v^3) =$