

Homework 11 Hexin Bi

1. $x^3 - 9x - 28 = 0$

$x = u + v$

$(u + v)^3 - 9(u + v) - 28 = 0$

$u^3 + 3u^2v + 3uv^2 + v^3 - 9(u + v) - 28 = 0$

$u^3 + v^3 + (3u^2v + 3uv^2 - 9(u + v)) - 28 = 0$

$u^3 + v^3 + (u + v)(3uv - 9) - 28 = 0$

$3uv - 9 = 0$

$u \cdot v = 3$

$u^3 + v^3 = 28$

$u^3 v^3 = 27$

$u = 1 \quad v = 3 \quad x = 4$

Roots are $x = 4 \quad x = -2 + \sqrt{3}i \quad x = -2 - \sqrt{3}i$

2. $x^3 - 30x - 133 = 0$

$x = u + v$

$(u + v)^3 - 30(u + v) - 133 = 0$

$u^3 + 3u^2v + 3uv^2 + v^3 - 30(u + v) - 133 = 0$

$u^3 + v^3 + (u + v)(3uv - 30) - 133 = 0$

$3uv - 30 = 0 \quad uv = 10$

$v^3 + u^3 = 133 \quad u^3 v^3 = 1000$

$u = 2 \quad v = 5 \quad x = 7$

Roots $x = 7 \quad x = -\frac{7}{2} + i\frac{\sqrt{3}}{2} \quad x = -\frac{7}{2} - i\frac{\sqrt{3}}{2}$

3. $x^3 + px + q = 0$

$x = (u + v)$

$(u + v)^3 + p(u + v) + q = 0$

$u^3 + 3u^2v + 3uv^2 + v^3 + p(u + v) + q = 0$



$$u^3 + v^3 + (u+v)(3uv+p) + q = 0$$

$$3uv = -p$$

$$u^3 + v^3 = q$$

$$u^3 v^3 = \left(\frac{p}{3}\right)^3$$

4. $x^3 + 3x^2 + 5x - 100 = 0$

Since general form is $x^3 + ax^2 + bx + c = 0$

$$a = 3 \quad b = 5 \quad c = -100$$

$$y = x + \frac{a}{3}$$

$$x = y - 1$$

$$(y-1)^3 + 3(y-1)^2 + 5(y-1) - 100 = 0$$

$$y^3 - 1 - 3y(y-1) + 3(y^2 - 2y) + 5y - 5 = 0$$

$$y = -3\sqrt[3]{\frac{-q}{3} + \sqrt{R}} + 3\sqrt[3]{\frac{-q}{3} - \sqrt{R}}$$

$$p = 3 \quad q = -103$$

$$R = \left(\frac{p}{3}\right)^3 + \left(\frac{q}{3}\right)^2$$

