Sarah Magno
Dr. Z, History of Math
10/17/21

Homework for Lecture 10 - OK to post

① Suppose that there are a finite number of primes, and call them $p_1, p_2, \ldots, p_k$. We use these to construct a new number, call it P, such that

$$P = p_1 p_2 \cdots p_k + 1$$

Therefore, this number is either prime (contradicting that the number of primes is finite) or it is composite. But it can't be divided by $p_1, p_2, \ldots, p_k$ since they all leave remainder 1 after division. So P must have a prime divisor that is different from $p_1, p_2, \ldots, p_k$, so we have shown that there exists a new prime number, which again contradicts that the list of primes is finite.

② $\boxed{2}$ $\boxed{3}$ ~~4~~ $\boxed{5}$ ~~6~~ $\boxed{7}$ ~~8~~ ~~9~~ ~~10~~ $\boxed{11}$ ~~12~~ $\boxed{13}$ ~~14~~ ~~15~~ ~~16~~ $\boxed{17}$ ~~18~~ $\boxed{19}$ ~~20~~ ~~21~~ ~~22~~
$\boxed{23}$ ~~24~~ ~~25~~ ~~26~~ ~~27~~ ~~28~~ $\boxed{29}$ ~~30~~ $\boxed{31}$ ~~32~~ ~~33~~ ~~34~~ ~~35~~ ~~36~~ $\boxed{37}$ ~~38~~ ~~39~~ ~~40~~ $\boxed{41}$ ~~42~~ $\boxed{43}$
~~44~~ ~~45~~ ~~46~~ $\boxed{47}$ ~~48~~ ~~49~~ ~~50~~ ~~51~~ ~~52~~ $\boxed{53}$ ~~54~~ ~~55~~ ~~56~~ ~~57~~ ~~58~~ $\boxed{59}$ ~~60~~ $\boxed{61}$ ~~62~~ ~~63~~ ~~64~~
~~65~~ ~~66~~ $\boxed{67}$ ~~68~~ ~~69~~ ~~70~~ $\boxed{71}$ ~~72~~ $\boxed{73}$ ~~74~~ ~~75~~ ~~76~~ ~~77~~ ~~78~~ $\boxed{79}$ ~~80~~ ~~81~~ ~~82~~ $\boxed{83}$ ~~84~~ ~~85~~
~~86~~ ~~87~~ ~~88~~ $\boxed{89}$ ~~90~~ ~~91~~ ~~92~~ ~~93~~ ~~94~~ ~~95~~ ~~96~~ $\boxed{97}$ ~~98~~ ~~99~~ ~~100~~ $\boxed{101}$ ~~102~~ $\boxed{103}$ ~~104~~ ~~105~~ ~~106~~
$\boxed{107}$ ~~108~~ $\boxed{109}$ ~~110~~ ~~111~~ ~~112~~ $\boxed{113}$ ~~114~~ ~~115~~ ~~116~~ ~~117~~ ~~118~~ ~~119~~ ~~120~~ ~~121~~ ~~122~~ ~~123~~ ~~124~~ ~~125~~ ~~126~~ $\boxed{127}$
~~128~~ ~~129~~ ~~130~~ $\boxed{131}$ ~~132~~ ~~133~~ ~~134~~ ~~135~~ ~~136~~ $\boxed{137}$ ~~138~~ $\boxed{139}$ ~~140~~

Step 1: cross out all multiples of 2 (except 2) → survivor: 3
Step 2: cross out all multiples of 3 (except 3) → survivor: 5
Step 3: cross out all multiples of 5 (except 5) → survivor: 7
Step 4: cross out all multiples of 7 (except 7) → survivor: 11
Step 5: cross out all multiples of 11 (except 11) → survivor: 13

The smallest survivor 13 is larger than $\sqrt{140}$, so we are done.

(3) 2 is not divisible by 3, but 3 is. 3^2 is not divisible by 3003.
$p = 3, a = 1$

$$L(3003) = [3,1]\ L(3003/3) = [3,1], L(1001)$$

The smallest prime that divides 1001 is 7. 1001 is not divisible by 7^2, so since $1001/7 = 143$,

$$L(1001) = [7,1], L(143)$$

143 is not divisible by 7, but it is divisible by 11. 143 is not divisible by 11^2, so $p = 11, a = 1$

$$L(143) = [11,1], L(13)$$

13 is not divisible by 11, but it is divisible by 13. 13 is not divisible by 13^2, so $p = 13, a = 1$

$$L(13) = [13,1], L(1)$$

Since $L(1)$ is the empty list, we work backward to obtain

$$L(13) = [13,1]$$
$$L(143) = [11,1], [13,1]$$
$$L(1001) = [7,1], [11,1], [13,1]$$
$$L(3003) = [3,1], [7,1], [11,1], [13,1]$$

Thus $3003 = 3 \cdot 7 \cdot 11 \cdot 13$

(4) We use the formula $\pi(n) = \frac{n}{\ln n}$, where $n = e^{100}$, to obtain $\frac{e^{100}}{\ln(e^{100})} =$

$\frac{e^{100}}{100(\ln e)} = \frac{e^{100}}{100}$, which is roughly the number of primes $\leq e^{100}$